Regularised approach to detection of constant modulus signals in MIMO channels

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A new suboptimal algorithm for detection of constant modulus signals in multiple-input multiple-output channels is presented. The deviation of the solution from the desired constant modulus property is used as a penalty or regularisation term in the conventional least squares cost function, and an iterative reweighted least squares procedure is used to minimise the regularised function.

Introduction: The maximum likelihood (ML) detector is the optimal algorithm in the sense of minimum probability of error for multiple-input multiple-output (MIMO) systems, but tends to be computationally intensive and impractical for many applications. This is true even for the sphere decoding (SD) algorithm of Fincke and Pohst [1], for which the expected complexity is polynomial over a wide range of signal-to-noise ratios (SNRs) and number of antennas [2]. Therefore, there is a strong demand for computationally efficient but suboptimal detection algorithms. Among them, probably the most popular is the vertical Bell Labs layered space time (V-BLAST) detector, which is a nulling-and-cancelling (NC) minimum mean-squared error (MMSE) detector with optimal ordering [3].

In this Letter, we present a new MIMO detection algorithm for constant modulus (CM) (e.g. binary/quadrature phase shift keying) signals. The deviation of the solution from the desired constant modulus property is used as a penalty or regularisation term in the conventional least squares cost function. Specifically, the penalty term uses an ϵ-insensitive quadratic loss function and, in this way, the problem turns out to be similar to a support vector regression (SVR) problem [4]. This loss function has been shown to have desirable properties (for instance, improved generalisation) for regression problems [4], and has been previously applied to blind equalisation with succesful results in [5] and [6]. To reduce the computational burden, the regularised cost function is minimised using an iterative reweighted least squares (IRWLS) procedure, which converges to the support vector machine solution. Some simulation results show that, for moderate or large-size problems, the proposed method provides better performance than the nulling-and-cancelling MMSE detector with much lower complexity than the sphere decoding algorithm.

Review of MIMO detection schemes: We consider a flat fading Rayleigh MIMO channel with n_T transmit antennas and $n_R \geq n_T$ receive antennas. This leads to the well-known baseband model

$$\boldsymbol{x} = \boldsymbol{H}\boldsymbol{s} + \boldsymbol{n} \qquad (1)$$

where $\boldsymbol{x}$ is the $n_R \times 1$ received vector, $\boldsymbol{s}$ is the $n_T \times 1$ transmitted vector, $\boldsymbol{n}$ is the $n_R \times 1$ noise vector and $\boldsymbol{H}$ is the $n_R \times n_T$ MIMO channel matrix. The channel $\boldsymbol{H}$ is perfectly known at the receiver and its entries are independent and identically distributed (i.i.d.) zero-mean complex Gaussian random variables with unit variance. On the other hand, the components of $\boldsymbol{n}$ are i.i.d. zero-mean, circularly symmetric complex Gaussian random variables with variance σ^2. Model (1) also encompasses frequency-selective MIMO channels as well as MIMO systems that use linear-dispersion codes; in these situations we have an $N \times M$ MIMO channel matrix with N, $M >> n_R$.

The optimal ML detector is the solution of the following integer least squares problem:

$$\hat{\boldsymbol{s}}_{ML} = \arg\min_{s \in D} \|\boldsymbol{x} - \boldsymbol{H}\boldsymbol{s}\|_2^2 \qquad (2)$$

where D denotes the set of all possible transmitted data vectors. This problem is known to be NP-hard and its computational complexity grows exponentially with the number of transmitting antennas n_T. To avoid the high computational cost of ML detection, several suboptimal procedures have been proposed. A simple detector consists in finding the MMSE estimate followed by a hard decision

$$\hat{\boldsymbol{s}}_{MMSE} = Q[(\boldsymbol{H}^H\boldsymbol{H} + \sigma^2\boldsymbol{I})^{-1}\boldsymbol{H}^H\boldsymbol{x}] \qquad (3)$$

where $Q[\cdot]$ denotes componentwise quantisation according to the symbol constellation used. Better results are obtained by the nulling-and-cancelling MMSE detector (denoted here as NC-MMSE), for which the initial MMSE symbol estimates are sorted according to the received SNR and the symbols are detected one by one. The contribution of each detected symbol is subtracted from the observations to reduce the interference.

MIMO detection of CM signals: In this Letter, we assume that the transmitted signal belongs to a constant modulus (CM) constellation $|s_i|^2 = 1$. This additional information can be exploited to regularise the unconstrained least squares cost function as follows:

$$J(s) = \frac{1}{2}\|\boldsymbol{x} - \boldsymbol{H}\boldsymbol{s}\|_2^2 + C\sum_{i=1}^{n_T} L_\epsilon(u_i) \qquad (4)$$

where C is a regularisation parameter, $L_\epsilon(\cdot)$ is a loss function that penalises the deviations from the desired CM property, and $u_i = |e_i|$, where $e_i = 1 - |s_i|^2$. Specifically, we use the following quadratic ϵ-insensitive loss function, which only penalises deviations larger than ϵ and has been successfully used in support vector regression problems [4–6]

$$L_\epsilon(u_i) = \begin{cases} 0, & u_i \leq \epsilon \\ (u_i - \epsilon)^2, & u_i > \epsilon \end{cases} \qquad (5)$$

Since the penalty term is a function of the modulus of the solution, the resulting regularised cost function is not convex and, therefore, an iterative procedure is needed to minimise (4). To this end, we first decompose e_i as $e_i = 1 - (s_i^{k-1})^* s_i$, where s_i^{k-1} is the soft estimate of the ith layer in the previous iteration. In this way, by considering now that s_i^{k-1} is fixed the minimisation of (4) becomes a quadratic programming (QP) problem [5]. To reduce the computational cost further, the QP problem is solved using an iterative reweighted least squares (IRWLS) procedure [6]. In particular, after the $(k-1)$th iteration a quadratic approximation of (4) is constructed as follows (a detailed derivation can be found in [6]):

$$J'(s) = \frac{1}{2}\|\boldsymbol{x} - \boldsymbol{H}\boldsymbol{s}\|_2^2 + \frac{1}{2}\sum_{i=1}^{n_T}(a_i e_i^* e_i) + \text{CTE} \qquad (6)$$

where CTE group constant terms that do not depend on $\boldsymbol{s}$ and the weights a_i are given by

$$a_i = \frac{C}{u_i^{k-1}} \frac{dL_\epsilon(u_i)}{du_i}\bigg|_{u_i^{k-1}} = \begin{cases} 0, & u_i \leq \epsilon \\ \dfrac{2C(u_i^{k-1} - \epsilon)}{u_i^{k-1}}, & u_i > \epsilon \end{cases}$$

Now, taking the derivative of (6) with respect to s and equating to zero, the following linear system of equations is obtained:

$$[\boldsymbol{H}^H\boldsymbol{H} + 2\boldsymbol{D}_a\boldsymbol{D}_{|s^{k-1}|^2}]s = \boldsymbol{H}^H\boldsymbol{x} + 2\boldsymbol{D}_a s^{k-1} \qquad (7)$$

where $\boldsymbol{D}_a = \text{diag}(a_1, \ldots, a_{nT})$ and $\boldsymbol{D}|s^{k-1}|^2 = \text{diag}(|s_1^{k-1}|^2, \ldots, |s_{n_T}^{k-1}|^2)$.

Similarly to [5, 6], to improve the convergence of the method the solution is smoothed as $s^k = \lambda s^{k-1} + (1-\lambda)s^*$, where s^* is the new solution of the system of equations and λ is a positive constant close to one. The procedure can be initialised using the soft estimate given by the unconstrained MMSE detector. To check the convergence of the procedure we evaluate the average modulus error (AME), which is defined as $AME(k) = \sqrt{(1/n_T)\sum_{i=1}^{n_T}(|s_i^k| - 1)^2}$. Finally, the proposed procedure is summarised in the following algorithm:

1. Initialise C, ϵ, δ and λ.
2. Obtain the initial MMSE solution: $s^0 = s_{MMSE}$.
3. WHILE convergence criterion not true.
   a. Calculate $\boldsymbol{D}_a$ and $\boldsymbol{D}_{|s^{k-1}|^2}$ and obtain s^* by solving (7).
   b. $s^k = \lambda s^{k-1} + (1-\lambda)s^*$.
   c. Check convergence criterion: $|AME(k) - AME(k-1)| \leq \delta$.
4. $\hat{\boldsymbol{s}} = Q[s^k]$.

Simulation results: We compare the bit error rate (BER) and the computational complexity of the MMSE detector, the NC-MMSE detector, the sphere decoding (SD) algorithm and the proposed detector (labelled in Figs. 1*a* and 1*b* as SVR-MMSE). For the

SVR-MMSE detector we choose $C = 7$, $\epsilon = 0.001$, $\lambda = 0.5$ and $\delta = 1e - 5$. Fig. 1*a* shows the BER for a 15×15 MIMO system and an 8-PSK constellation. For the same scenario Table 1 compares the average execution time in milliseconds of the four algorithms programmed in Matlab and running on a PC Pentium IV (2.8 GHz). In comparison to the NC-MMSE, the proposed scheme saves up to 2 dB of E_b/N_0 with only a moderate increase in computational cost, which is still much lower than that of the SD algorithm. Fig. 1*b* shows the BER for a 45×45 MIMO channel with an 8-PSK constellation. For this example the results of the SD algorithm are not given due to its high computational cost. Now, the improvement over the NC-MMSE is close to 4 dB of E_b/N_0.

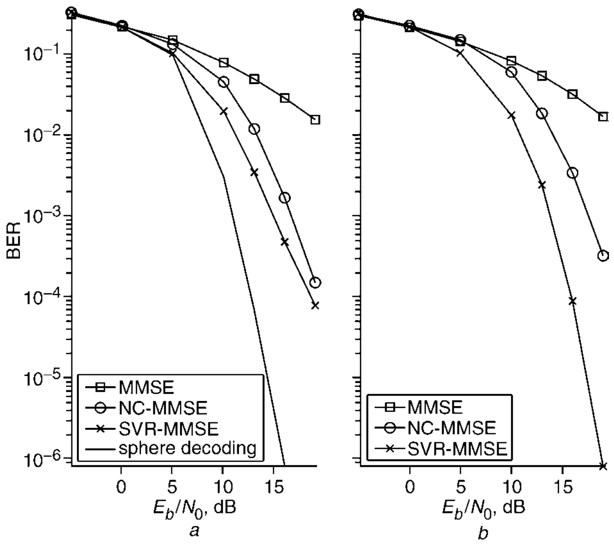

Fig. 1 *BER against E_b/N_0*

a 15×15 MIMO channel
b 45×45 MIMO channel

Table 1: Computational cost (average execution time in milliseconds) for SD, MMSE, NC-MMSE and SVR-MMSE detector for 15×15 MIMO channel

E_b/N_0 (dB)	5	10	13	16	19
MMSE	0.47	0.41	0.46	0.47	0.47
NC-MMSE	3.90	3.41	3.71	3.72	3.71
SVR-MMSE	23.44	22.00	20.07	18.64	17.28
SD	107 000	3799.90	558.31	221.76	167.06

Conclusions: We have developed a suboptimal method for detection of CM signals in MIMO systems. The method efficiently exploits the additional information provided by the CM alphabet property. In terms of BER and computational cost the proposed detector provides intermediate results between the NC-MMSE detector and the sphere decoding algorithm, mainly for MIMO systems of large or moderate size.

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