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MODELING PHONE CALL DURATIONS VIA SWITCHING POISSON PROCESSES WITH APPLICATIONS IN MENTAL HEALTH

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ABSTRACT

This work models phone call durations via switching Poisson point processes. This kind of processes is composed by two intertwined intensity functions: one models the start of a call, whereas the other one models when the call ends. Thus, the call duration is obtained from the inverse of the intensity function of finishing a call. Additionally, to model the circadian rhythm present in human behavior, we shall use a (positive) truncated Fourier series as the parametric form of the intensities. Finally, the maximum likelihood estimates of the intensity functions are obtained using a trust region method and the performance is evaluated on synthetic and real data, showing good results.

Index Terms— Intensity function, maximum likelihood (ML) estimation, point processes, switching Poissing process, trust region method

1. INTRODUCTION

Modeling event durations is an overarching goal in many fields, which has important industrial, economical and medical applications. Recently, with the widespread use of mobile phones, there is an emerging need for such techniques for the automatic assessment of social activity, which is especially relevant in psychiatry. Indeed, psychiatrists lack objective tools to monitor the health condition of patients in between medical consultations [1]. These tools would allow to predict relapses [2], thereby improving the patients' quality of life and reducing the high treatment costs [3]. Therefore, there is an intense research effort to capture the interactions of patients with their electronic devices (like their smartphones) and subsequently apply signal processing techniques to obtain a proxy of their health condition. In particular, notice that electronic interactions via phone calls can be taken as indicators of the actual social activity [4], and the durations thereof are strongly sensitive to the kind of relationship between the speakers [5]. Hence, in this work, we propose a novel framework to model phone call durations, which can be used as an integral measure of the well-being and social interactions of psychiatric patients.

There are studies that have used the call duration of psychiatric patients in order to assess their mental condition, i.e., [6]. However, the features that are used to model them are rather daunting, since they are basically the overall time making all the calls in a day, and this ignores the temporal structure, which can be important. To the best of our knowledge, the only approach to obtain such temporal structure has been to compute the mean call duration per time slot, like in [5]. On the other hand, the problem of modeling the durations of other events related to human activities has been addressed in [7], where the state transition of a Markov model is used to model the time that a person spends in different parts of a house. This Markovian approach has been prominent in many diverse fields, and it has been used to model the duration of, e.g.: phonemes [8], transmission signals [9], and the time to perform a given task in a project [10].

Despite the success of the Markovian modeling, we must point out that it is not adequate for modeling phone call durations due to their particular features. Specifically, phone calls produce a series of point observations: every time that a phone call is started and once it is accomplished. In addition, both the time when the call starts and the distance between those two timestamps (i.e., the duration of the call) may vary, for instance, depending on the hour of the day. Hence, the correct framework to model those durations are point processes [11] and, in particular, we shall use non-homogeneous Poisson processes.

Point processes can be formulated to explicitly model event durations by considering a marked process [11]. That is, every time a phone call is made, a mark with the call duration is added, and this mark is modeled by a probability density function. Nevertheless, this approach may assign non-zero probability of starting a new call whilst one is still on course, which is an undesirable feature. One model that prevents such a problem is the *switched Poisson* process [12],

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which was proposed to estimate the waiting time of a server queue system with alternating varying Poisson rates. Other alternative is to consider a single parametric form of a non-homogeneous Poisson process whose parameters change at different change points. By inferring those change points and their time span, it is possible to estimate the duration of different events. This technique was applied to estimate the duration of different drought periods in Brazil [13] and ozone threshold exceedances in Mexico City [14].

The idea of a single Poisson model whose parameters change at given points, which define some intervals, has been furthered leveraged by adding a Markovian structure to the switching times of the Poisson process. This model, known as the *Markov-modulated Poisson* process, has been used for expediting detections in surveys of marine mammal abundance [15], and to model inter-trade durations in financial markers [16]. A noteworthy case happens when the Markov transition matrix has only two hidden states, like in [17]. Actually, this special case has been revisited in [18], where the authors describe an complementary derivation of the process that considers two (conditional) intensities, each of which representing the instantaneous hazard of going from one state to the other. Hence, when one of the intensities is active, the other is null. This formulation has a drawback, namely, it does not allow to explicitly infer the overall intensities of the process, since they are always conditioned on some realizations, which determine which intensity is active at each time.

To overcome this problem, in this paper, we extend the work of [18], by providing an alternative derivation of the process which also decouples the overall intensities from the switching mechanism that makes one of them null at given times according to the particular realization of the dataset. Finally, since we are interested in psychiatric applications, we need to consider the circadian rhythm [19], and we therefore propose to use a (non-negative) truncated Fourier series expansion as the parametric form of the (conditional) intensity functions.

2. MODELING PHONE CALL DURATIONS

2.1. Switching Poisson

Let $\tilde{\lambda}_c^*(t)$ be the *overall* intensity function of making a call, and that of finishing it (i.e., the one that determines its duration) be $\tilde{\lambda}_d^*(t)$. When a particular realization of the process is observed, the intensity of making a call, denoted as $\lambda_c^*(t)$, and that of finishing it, $\lambda_d^*(t)$, become intertwined. That is, when one of them is non-zero, the other one must be null, and vice versa, in order to comply with some empirical limitations, such as the fact that a call cannot be finished before it is started. Moreover, we shall consider the most probable case in which the patient is not making a call neither at the beginning nor at end of the observation period, but it is straightforward

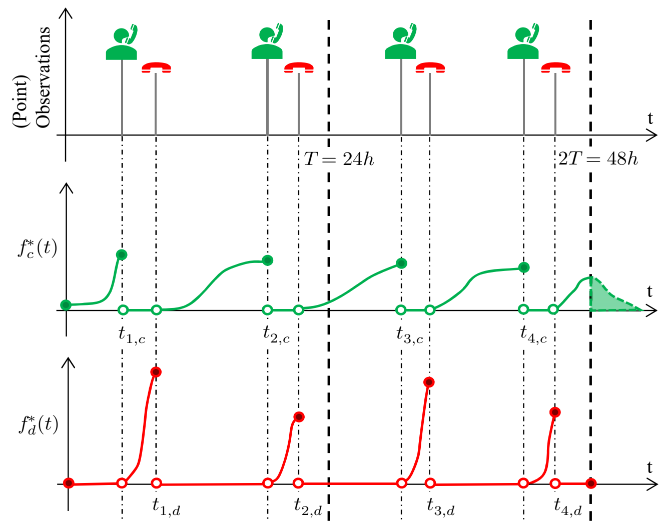

Fig. 1. Probability density functions of the switching Poisson process for an example where $T_{\text{obs}} = 48h$.

to modify the model for the three remaining cases. Then, the intensities are related as follows

$$\lambda_c^*(t) = \tilde{\lambda}_c^*(t)\Big[N_d(t) - N_c(t) + 1\Big], \qquad (1)$$

and

$$\lambda_d^*(t) = \tilde{\lambda}_d^*(t)\Big[N_c(t) - N_d(t)\Big],$$

where $N_c(t)$ and $N_d(t)$ represent, respectively, the number of times that a patient has started and finished a call up to time t. Since this formulation switches on and off two independent non-homogeneous Poisson processes, we shall refer to it as the *switching Poisson* process.

In the following, we derive the maximum likelihood (ML) estimates of the foregoing intensities. Before proceeding, let us denote the (conditional) probability density of making a call (conversely finishing it) as $f_c^*(t)$ (conversely $f_d^*(t)$), and let $F_c^*(t)$ denote the (conditional) cumulative distribution function of $f_c^*(t)$. Further, let $t_{i,c}$ be the timestamps recorded when a call is started, and $t_{i,d}$ be the timestamps obtained when a call is completed. Then, as shown in Fig. 1, it follows that the likelihood function of a switching Poisson process that has seen n calls over an observation period T_{obs} is

$$\mathcal{L} = \left(\prod_{i=1}^{n} f_c^*(t_{i,c})\right)\left(1 - F_c^*(T_{\text{obs}})\right)\left(\prod_{i=1}^{n} f_d^*(t_{i,d})\right),$$

where, for the lack of notation, we use the so-called "empty product", which is 1, by definition, in the absence of observations. Notice that the middle term comes from the fact that the unobserved event $t_{n+1,c}$ must occur after the end of the observation period. In order to work with a more manageable expression, we will use intensity functions instead of density functions as in [20], which are related by

$$f^*(t) = \lambda^*(t) e^{-\int_{t_i}^{t} \lambda^*(s) ds}, \qquad (2)$$

where $\lambda^*(t)$ is the intensity function and t_i denotes the timestamp of the last observed event that is being analyzed (either the beginning or the end of a call). Now, using (2), the second term in $\mathcal{L}$ can be rewritten as

$$\begin{aligned}1 - F_c^*(T_{\text{obs}}) &= e^{-\int_{t_{n,c}}^{t_{n,d}} \lambda_c^*(s)ds - \int_{t_{n,d}}^{T_{\text{obs}}} \lambda_c^*(s)ds} \\ &= e^{-\int_{t_{n,d}}^{T_{\text{obs}}} \tilde{\lambda}_c^*(s)ds},\end{aligned}$$

since $\lambda_c^*(t) = 0, \ \forall t \in [t_{i,c}, t_{i,d})$, and $\lambda_c^*(s)$ has been replaced by $\tilde{\lambda}_c^*(s)$ in accordance with (1). Similar arguments can be employed upon the first and third terms. For instance, defining $t_{0,c} = t_{0,d} = 0$, we may write the first term in $\mathcal{L}$ as

$$\begin{aligned}\prod_{i=1}^{n} f_c^*(t_{i,c}) &= \prod_{i=1}^{n} \lambda_c^*(t_{i,c}) e^{-\int_{t_{(i-1),c}}^{t_{i,c}} \lambda_c^*(s)ds} \\ &= \prod_{i=1}^{n} \tilde{\lambda}_c^*(t_{i,c}) e^{-\int_{t_{(i-1),d}}^{t_{i,d}} \tilde{\lambda}_c^*(s)ds}.\end{aligned}$$

Now, taking the logarithm and defining $\mathbb{T}_c$ as the set of time instances when someone is waiting to make a call, and $\mathbb{T}_d$ as the time span when calls are on course, the ML estimation problem can be stated as the minimization of

$$-\sum_{i=1}^{n} \log\left(\tilde{\lambda}_c^*(t_{i,c})\right) - \sum_{i=1}^{n} \log\left(\tilde{\lambda}_d^*(t_{i,d})\right) + \int_{\mathbb{T}_c} \tilde{\lambda}_c^*(s)\, ds + \int_{\mathbb{T}_d} \tilde{\lambda}_d^*(s)\, ds,$$

which shows that the optimization of the two intensities is independent from one another.

To model the evolution of call durations over time, the *inverse* of the intensity of finishing a call, $\tilde{\lambda}_d^*(t)$, should be used, as it can be interpreted as the mean call duration. Nonetheless, this raw estimation could not be suitable for those regions where $\tilde{\lambda}_d^*(t) \approx 0$, since its inverse would blow up. To avoid this issue, the problem can be regularized so that the aforementioned intensity is pushed towards a value ρ using the regularizer $R\,\tilde{\lambda}_d^*(t)$. A natural choice for ρ is the inverse of the average call duration, which yields an inverse of $\tilde{\lambda}_d^*(t)$ that oscillates around such average. Hence, the regularized ML estimation of $\tilde{\lambda}_d^*(t)$, with penalization parameter μ, boils down to the minimization of

$$-\sum_{i=1}^{n} \log\left(\tilde{\lambda}_d^*(t_{i,d})\right) + \int_{\mathbb{T}_d} \tilde{\lambda}_d^*(s)\, ds + \mu \underbrace{\int \left(\tilde{\lambda}_d^*(t) - \rho\right)^2 dt}_{R\,\tilde{\lambda}_d^*(t)}. \quad (3)$$

One final comment is in order. As can be seen, the estimation of $\tilde{\lambda}_c^*(t)$ can be ignored if the aim is just to model call durations, but this switching Poisson framework allows us to obtain the intensity of making a call as a by-product. Thus, hereafter, we will only consider the problem in (3).

2.2. A parametric form for the intensity function

In this section, we provide an efficient way of solving the ML estimation problem in (3), when using a parametric form suitable for the application at hand. Concretely, to model the circadian rhythm, present in phone calls, we propose to use a truncated Fourier series with period $T = 24$ hours, that is

$$\tilde{\lambda}_d^*(t) = \frac{a_0}{2} + \sum_{k=1}^{K} \left[a_k \cos\left(\frac{2\pi k}{T} t\right) + b_k \sin\left(\frac{2\pi k}{T} t\right) \right].$$

Needless to say, the intensity must be non-negative. Thus, to ensure the non-negativeness of $\tilde{\lambda}_d^*(t)$, the Fourier coefficients are computed as [21, 22]

$$a_k = 2\mathbf{c}_r^T \mathbf{U}_k \mathbf{c}_r + 2\mathbf{c}_i^T \mathbf{U}_k \mathbf{c}_i, \quad b_k = 2\mathbf{c}_r^T \left(\mathbf{U}_k - \mathbf{U}_k^T\right) \mathbf{c}_i,$$

where $\mathbf{c}_r = \Re(\mathbf{c})$ is the real part of $\mathbf{c}$, $\mathbf{c}_i = \Im(\mathbf{c})$ its imaginary part, $\mathbf{c} = [c_0, \ldots, c_K]^T \in \mathbb{C}^{K+1}$, and $\mathbf{U}_k \in \mathbb{R}^{(K+1)\times(K+1)}$ is a Toeplitz matrix whose entries on the kth diagonal are 1, and 0 elsewhere.

Using the above quadratic forms for the coefficients, $\tilde{\lambda}_d^*(t)$ can be succinctly expressed as

$$\tilde{\lambda}_d^*(t) = \mathbf{d}^T \mathbf{T}(t) \mathbf{d},$$

where $\mathbf{d} = [\mathbf{c}_r^T, \mathbf{c}_i^T]^T$, and the matrix $\mathbf{T}(t)$ (not to be confused with the period $T = 24$ hours) is given by

$$\mathbf{T}(t) = \begin{bmatrix} \mathbf{C}(t) & \mathbf{S}(t) \\ \mathbf{S}^T(t) & \mathbf{C}(t) \end{bmatrix},$$

with

$$\mathbf{C}(t) = \mathbf{I}_{K+1} + \sum_{k=1}^{K} \cos\left(\frac{2\pi k}{T} t\right) \left(\mathbf{U}_k + \mathbf{U}_k^T\right),$$

and

$$\mathbf{S}(t) = \sum_{k=1}^{K} \sin\left(\frac{2\pi k}{T} t\right) \left(\mathbf{U}_k - \mathbf{U}_k^T\right).$$

Moreover, using the properties of the Kronecker product and those of the vec operator, the regularizer admits the compact form

$$R\,\tilde{\lambda}_d^*(t) = (\mathbf{d} \otimes \mathbf{d})^T\, \overline{\mathbf{Q}}\, (\mathbf{d} \otimes \mathbf{d}) + \rho^2 T - 2\rho T \mathbf{d}^T \mathbf{d},$$

where

$$\overline{\mathbf{Q}} = \int_0^T \operatorname{vec}\left(\mathbf{T}(t)\right) \left(\operatorname{vec}\left(\mathbf{T}(t)\right)\right)^T dt,$$

and vec is the vectorization operator. Defining $\mathbf{I}$ as the identity matrix of dimension $2K+2$, the regularized ML problem in (3) can be expressed as the minimization of the cost function

$$J_{\text{reg}}(\mathbf{d}) = \mathbf{d}^T \left(\overline{\mathbf{T}}_{\mathbb{T}_d} - 2\mu\rho T \mathbf{I}\right) \mathbf{d} - \sum_{i=1}^{n} \left(\log\left(\mathbf{d}^T \mathbf{T}(t_{i,d}) \mathbf{d}\right) \right) + \mu\, (\mathbf{d} \otimes \mathbf{d})^T \overline{\mathbf{Q}} (\mathbf{d} \otimes \mathbf{d}), \quad (4)$$

Overall call rate in calls/h ($\times 10^{-3}$)	Mean Number of observed calls	AIC		BIC	
		$P(K=\widehat{K})$	NMSE (dB)	$P(K=\widehat{K})$	NMSE (dB)
84	59.246	0.687	-3.2330	0.354	-0.4088
124	86.626	0.803	-4.1876	0.529	-0.5374
164	114.322	0.848	-5.2495	0.673	-1.1821
204	142.151	0.856	-5.7202	0.746	-1.9765
244	169.311	0.859	-6.3645	0.814	-2.7231
284	197.685	0.861	-6.7828	0.886	-4.0008
324	224.759	0.859	-7.5082	0.904	-4.9020
364	250.744	0.854	-8.0443	0.945	-6.3397

Table 1. Synthetic data results.

where

$$\overline{\mathbf{T}}_{\mathbb{T}_d} = \int_{\mathbb{T}_d} \mathbf{T}(t)\, dt.$$

It can readily be shown that the cost function in (4) is non-convex [23], which requires the use of non-convex optimization techniques. In particular, we propose to employ the trust-region method described in [24], which needs the gradient and the Hessian of $J_{\text{reg}}(\mathbf{d})$. Using matrix differential calculus [25], the gradient becomes

$$\nabla_{\mathbf{d}} J_{\text{reg}}(\mathbf{d}) = 2\left(\overline{\mathbf{T}}_{\mathbb{T}_d} - 2\mu\rho T\mathbf{I}\right)\mathbf{d} - 2\sum_{i=1}^{n}\left(\frac{\mathbf{T}(t_{i,d})\mathbf{d}}{\mathbf{d}^T\mathbf{T}(t_{i,d})\mathbf{d}}\right) + 2\mu\left(\mathbf{I}\otimes\mathbf{d}^T + \mathbf{d}^T\otimes\mathbf{I}\right)\overline{\mathbf{Q}}\left(\mathbf{d}\otimes\mathbf{d}\right),$$

and the Hessian is

$$\mathbf{H}J_{\text{reg}}(\mathbf{d}) = 2\left(\overline{\mathbf{T}}_{\mathbb{T}_d} - 2\mu\rho T\mathbf{I}\right) - \sum_{i=1}^{n}\left[\frac{2}{\mathbf{d}^T\mathbf{T}(t_{i,d})\mathbf{d}}\mathbf{T}(t_{i,d}) - \left(\frac{2}{\mathbf{d}^T\mathbf{T}(t_{i,d})\mathbf{d}}\right)^2 \mathbf{T}(t_{i,d})\mathbf{d}\mathbf{d}^T\mathbf{T}(t_{i,d})\right] + 2\mu(\mathbf{I}\otimes\mathbf{d} + \mathbf{d}\otimes\mathbf{I})^T\,\overline{\mathbf{Q}}\,(\mathbf{I}\otimes\mathbf{d} + \mathbf{d}\otimes\mathbf{I}) + 2\mu\left[\left((\mathbf{d}\otimes\mathbf{d})^T\,\overline{\mathbf{Q}}\right)\otimes\mathbf{I}\right]\mathbf{J}\left[\text{vec}\left(\mathbf{I}\right)\otimes\mathbf{I}\right],$$

where

$$\mathbf{J} = (\mathbf{I}\otimes\mathbf{K}_{2K+2,2K+2}) + \mathbf{K}_{2K+2,(2K+2)^2},$$

and $\mathbf{K}_{p,q}$ is the commutation matrix [26].

3. RESULTS

In this section, the performance of the switching Poisson process for modeling phone call durations of psychiatric patients is assessed. To validate the inference method, we consider an experiment with synthetic data. Notice that in order to correctly analyze the suitability of the procedure for the target application, it is important to generate data around the expected regime. Specifically, using the call data of 259 psychiatric patients in [21], it is found that the average call duration and rate are 167.537 s, and 0.244 calls/h, respectively. Thus, we design the experiment so that the (non-negative) Fourier series

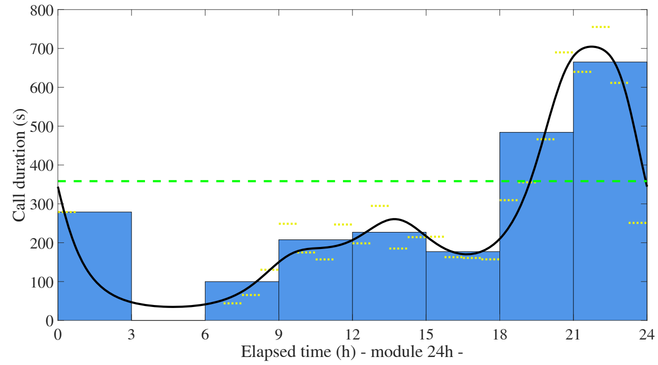

Fig. 2. Estimated call durations of a patient diagnosed with an F3 disease.

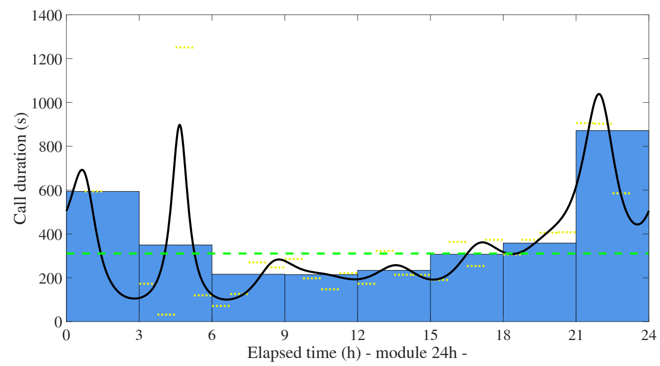

Fig. 3. Estimated call durations of a patient diagnosed with an F5 disease.

associated to $\tilde{\lambda}_d^*(t)$ produces phone calls around the aforementioned duration. In order to do so, it is useful to notice that

$$\frac{1}{T}\int_0^T \tilde{\lambda}_d^*(t)\, dt = \|\mathbf{c}\|^2. \tag{5}$$

Hence, we propose to sample $\mathbf{c}$ from a zero-mean Gaussian distribution with identity covariance matrix and then normalize it appropriately to control the mean of the randomly generated Fourier series. This determines, in turn, the mean of the calls that the intensity will produce when sampling from it.

Similarly, $\tilde{\lambda}_c^*(t)$ is also parametrized by a randomly generated (non-negative) Fourier series, but (5) is now used to sweep the mean call rate associated to those intensities over the value found in the aforesaid dataset. In this way, fixing $T_{\text{obs}} = 30$ days, it is possible to indirectly control the number of observations. In our experiment, $K = 3$ and two information theoretic criteria were considered to determine it, namely the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) [27].

The results are presented in Table 1, where the probability of detection of K, i.e., $P(K = \hat{K})$, is shown for 1 000 synthetic datasets generated from the different intensity pairs. This table also shows the normalized mean square error (NMSE) between the intensity of ending a call selected by each of the order selectors, and the one that generated the data. In particular, it is computed as the integral of the squared difference between the two intensities, divided by the integral of the square of the synthetic intensity. Those integrals are evaluated in one period. Lastly, it must be mentioned that several experiments have shown that a value of the regularization strength of $\mu = 4 \cdot 10^{-5}$ produces desirable results over a wide range of situations, hence, during our analysis, we pick that value. As can be seen, the AIC offers suitable and superior performance metrics compared to those of BIC, so it is the one that is selected for the subsequent analysis of real data.

In Fig. 2, we show the results of the switching Poisson process for the estimation of the 937 phone call durations recorded during 302 days of a thirty-four-year-old female psychiatric patient diagnosed with an F3 disease, according to the ICD-10 codes [28]. The solid line of this figure corresponds with the inverse of the estimated $\tilde{\lambda}_d^*(t)$ with $\hat{K} = 4$, according to the AIC. Additionally, following the current approach, the bars represent the mean duration of all the calls contained in time slots of 3 hours, what provides a coarse empirical estimate. On the other hand, the dotted yellows lines also represent the mean duration of all the observed calls in the designated slots, but since they span 45 min, they yield finer, yet noisier estimates. As depicted in the figure, no call duration was observed at all from 0:45 to 6:45 A.M., but the model is robust enough to handle this inimical situation. Finally, the dashed green line depicts the mean of all the observed call durations.

Fig. 2 reveals that the current approaches to model call durations of patients, such as summing the duration of all the calls in the same day, can be overly simplistic, since there is important information about the temporal dynamics of the circadian rhythm that can be neglected. For instance, the proposed model indicates that the patient has the shortest phone conversations from 6 to 9 A.M., probably because it coincides with the time she and her contacts wake up. Then, the duration of the calls increases almost 3-fold from 9 to 15, what could indicate that she is making some calls for work. From 3 to 6 P.M. the duration of the phone calls is shortened, which may coincide with the time to commute to her house or lunchtime in Spain. By the end of the evening, the duration of the calls increases up to 7 times compared to the 6 to 9 A.M. period, and it is likely to be the time she has long emotional conversations with her friends. Finally, she goes to bed, thereby not making any call during the night. Furthermore, Fig. 3 shows the analysis of the 1 179 phone call durations observed over 208 days of a female patient diagnosed with an F5 disease and aged 53 years. Since it is more complex, the model selects $\hat{K} = 8$, and it reveals a nocturnal pattern with two peaks of activity during the night.

4. CONCLUSION AND FUTURE WORK

In this paper, we have analyzed the feasibility of the switching Poisson process to model phone call durations, which can be applied to objectively measure the social activity of psychiatric patients. This process makes use of two intertwined, yet independent intensities: one that models the beginning, and other one that models the end of calls. Thus, only one intensity may be active at a given time when phone calls are observed. With this formulation, the expected call duration can readily be obtained by taking the inverse of the intensity describing the end of the calls. Moreover, to capture the inherent circadian rhythm, we have parametrized the intensities by (non-negative) Fourier series expansions. To make the model useful in situations where the intensity of finishing a call is close to zero, and hence its inverse may blow up, we have proposed a regularized formulation. We have evaluated the algorithm upon synthetic data, taking into account the characteristics of the intensity functions that are present in real data, and we have shown two examples of the method unveiling the circadian rhythm of real patients, which shows good results.

In the future, it would be of great interest to enrich the model so that it makes use of complementary information, such as the caller or the type of call. It would also be useful to consider mixture models for the intensity functions, in such a way that several patterns of the phone call durations can be extracted from the same patient. In this way, it would be possible to account for some external factors, such as the fact that a phone call is performed on a weekend or weekday.

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