Spatial rank estimation in Cognitive Radio Networks with uncalibrated multiple antennas

Gonzalo Vazquez-Vilar
Dept. of Signal Theory and Communications
University of Vigo
36310 Vigo, Spain
gvazquez@gts.uvigo.es

David Ramírez
Dept. of Electrical Engineering and Information Technology
Universität Paderborn
33098 Paderborn, Germany
david.ramirez@sst.upb.de

Roberto López-Valcarce
Dept. of Signal Theory and Communications
University of Vigo
36310 Vigo, Spain
valcarce@gts.uvigo.es

Javier Vía
Dept. of Communications Engineering
University of Cantabria
39005 Santander, Spain
jvia@gtas.dicom.unican.es

Ignacio Santamaría
Dept. of Communications Engineering
University of Cantabria
39005 Santander, Spain
nacho@gtas.dicom.unican.es

ABSTRACT

Spectrum sensing is a key component of the Cognitive Radio paradigm. Multiantenna detectors can exploit different spatial features of primary signals in order to boost detection performance and robustness in very low signal-to-noise ratios. However, in several cases these detectors require additional information, such as the rank of the spatial covariance matrix of the received signal. In this work we study the problem of estimating this rank under Gaussianity assumption using an uncalibrated receiver, i.e. with different (unknown) noise levels at each of the antennas.

Categories and Subject Descriptors

C.2.1 [**Network Architecture and Design**]: Wireless communication

Keywords

Cognitive Radio, Spectrum Sensing, Multiantenna, Rank Estimation, ML

1. INTRODUCTION

Cognitive Radio (CR) has gained popularity as a means to alleviate the apparent scarcity of spectral resources as seen today [10, 11]. The key idea behind CR is to allow opportunistic access to temporally and/or geographically unused licensed bands. In order to keep the interference produced to licensed (primary) users at sufficiently low levels, novel powerful detection schemes are required [2].

Recent developments show the advantages of exploiting the spatial properties of the received primary signal in order to boost both robustness and detection performance when multiple antennas are available. The detection schemes proposed in [15, 17] exploit the fact that the received signal presents a spatial rank $P = 1$ with noises assumed independent identically distributed (iid) across the antennas. When the noises are assumed uncorrelated non-iid detectors have been proposed in [3, 9]. These detectors either assume rank-1 primary signals or unstructured signals. The case of (known) rank-$P > 1$ was treated in [19, 12, 13], under the assumption of both iid and non-iid noises.

Note that the detection schemes above have been derived under different assumptions on the spatial rank of the received signal covariance matrix. The reason is that, in practical scenarios, the spatial rank of the received signals may be larger than one. This is the case, for example, if multiple independent users (e.g. from adjacent cells) simultaneously access the same frequency channel. Alternatively, many state-of-the-art communication standards consider the simultaneous transmission of different data streams through multiple antennas to achieve multiplexing gain and/or the use of space-time codes to enhance spatial diversity. For these systems, the signal received at the multiantenna sensor will exhibit a spatial rank equal to the number of independent streams or the spatial size of the code, respectively. Examples range from broadcasting standards, such as the european DVB-T2 [6] which considers 2-antenna space-time Alamouti codes, to point-to-multipoint standards, such as IEEE 802.11n [7], IEEE 802.16 [8] or LTE [1], which support up to four transmit antennas.

In practice, however, the spatial rank cannot be assumed known a priori since it may depend on the current state of the primary network. Hence, it is of interest to develop detection schemes robust to uncertainties on the spatial rank P of the primary signal. In this work we address the estimation of the rank of primary signal spatial covariance as a first step towards the derivation of detection schemes robust to rank uncertainty.

This problem has been extensively addressed in the literature in the context of model selection. In [18] the authors apply the Akaike information criterion (AIC) and the minimum description length (MDL) criterion in order to obtain a model selection framework. Other popular metric is the Bayesian information criterion (BIC) first proposed in [14].

Recently, in [5] Chiani et al. applied both AIC and BIC to the problem of estimating the number of signals in cognitive radio networks from short data records when the noises across the antennas are assumed iid. In several cases, however, this assumption may not completely hold. For example, tolerances in the components of the analog frontends at different antennas will result in deviations of the noise level from antenna to antenna, and as it turns out, the schemes proposed in [18] and [5] are very sensitive to these calibration errors.

Here we propose a spatial rank estimator which can deal with uncalibrated multiantenna receivers, build upon the ML estimation scheme proposed in [12]. The paper is organized as follows. In Section 2 we present the system model and formulate the problem. In Section 3 we present the general problem of of rank estimation based on information criteria. A numerical analysis of the performance of the proposed scheme is included in Section 4 and Section 5 closes this paper with some concluding remarks.

2. PROBLEM FORMULATION

Consider a spectrum monitor equipped with M antennas which is to monitor a given frequency channel. The received signals are downconverted and sampled at the Nyquist rate. We assume no synchronization with any potentially present primary signal. Primary transmission, if present, has an (unknown) spatial rank P, and for tractability a frequency-flat channel is assumed.

The spectrum monitor acquires K samples from the mth antenna arranged in a $K\times 1$ vector $\mathbf{y}_m$, which can be written as

$$\mathbf{y}_m = \mathbf{S}\mathbf{h}_m + \sigma_m \mathbf{n}_m, \tag{1}$$

where the $K \times P$ matrix $\mathbf{S} \doteq [\mathbf{s}_1\ \mathbf{s}_2 \cdots \mathbf{s}_P]$ is comprised of P primary signal streams, $\mathbf{h}_m$ denotes the $P\times 1$ channel vector from the primary system to the mth receiver antenna, σ_m^2 denotes the noise power, and $\mathbf{n}_1, \mathbf{n}_2, \ldots, \mathbf{n}_M$ represent the the $K \times 1$ independent noise streams.

Additionally we assume that both primary streams and noise processes are temporally white with unit variance so that

$$\mathrm{E}[\mathbf{s}_p \mathbf{s}_q^H] = \mathrm{E}[\mathbf{n}_p \mathbf{n}_q^H] = \begin{cases} \mathbf{I}_K & \text{if } p = q, \\ \mathbf{0} & \text{if } p \neq q, \end{cases} \tag{2}$$

Note that the model above assumes that the channel from the primary transmitter to the spectrum monitor is frequency-flat in the RF channel bandwidth, and that it remains constant for the duration of the sensing time. We restrict our analysis to both signal and noise following a zero-mean Gaussian distribution. The reasons for adopting a Gaussian model for the primary signal are, (i) the Gaussian pdf for a signal is the least informative one when the noise is assumed Gaussian as well, (ii) the Gaussian model is accurate for primary system using multicarrier modulation when the number of subcarriers is large enough [16], (iii) this model is tractable and Gaussianity is a common assumption in the development of signal detectors, either explicitly or implicitly.

The received signal can be compactly written in matrix form as

$$\mathbf{Y} = \mathbf{S}\mathbf{H} + \mathbf{N}\boldsymbol{\Sigma}, \tag{3}$$

where we have defined the $K\times M$ received signal matrix $\mathbf{Y} \doteq [\mathbf{y}_1\ \mathbf{y}_2 \cdots \mathbf{y}_M]$, the $P\times M$ channel matrix $\mathbf{H} \doteq [\mathbf{h}_1\ \mathbf{h}_2 \cdots \mathbf{h}_M]$, the $M \times M$ noise levels diagonal matrix given by $\boldsymbol{\Sigma} = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_M)$ and the $K \times M$ noise samples matrix $\mathbf{N} \doteq [\mathbf{n}_1\ \mathbf{n}_2 \cdots \mathbf{n}_M]$.

The matrix $\mathbf{Y}$ can be described as a complex Gaussian random matrix with iid rows, each of them with the same covariance matrix

$$\mathbf{R} \doteq \mathrm{E}\left[\frac{\mathbf{Y}^H\mathbf{Y}}{K}\right] = \mathbf{H}^H\mathbf{H} + \boldsymbol{\Sigma}^2. \tag{4}$$

In the presented model, the receiver has no knowledge about P, $\mathbf{H}$, or the noise covariance matrix $\boldsymbol{\Sigma}^2$. The problem we are interested in is to find a good estimator of the primary signal rank P from the received data matrix $\mathbf{Y}$.

Note that since both noise and signal are assumed temporally iid, the likelihood is given by the product of the individual pdfs, i.e.,

$$f(\mathbf{Y}\,|\,\mathbf{R}) = \frac{1}{\pi^{MK}\det(\mathbf{R})^K} \exp\left\{-K \,\mathrm{tr}\left(\mathbf{C}\mathbf{R}^{-1}\right)\right\}, \tag{5}$$

where

$$\mathbf{C} \doteq \frac{1}{K}\mathbf{Y}^H\mathbf{Y} \tag{6}$$

denotes the sample covariance matrix. Since the data matrix $\mathbf{Y}$ appears in (5) only through $\mathbf{C}$, the sample covariance matrix $\mathbf{C}$ is a sufficient statistic for the estimation of the unknown parameters.

3. RANK ESTIMATION BASED ON INFORMATION CRITERIA

In general the problem of rank estimation can be presented in the folowing form [5]

$$\begin{aligned} \hat{P} &= \arg\min_P \left\{-\log\left(\max_{\mathbf{R}|P} f(\mathbf{Y}\,|\,\mathbf{R})\right) + L(\nu(P), K)\right\} && (7) \\ &= \arg\min_P \left\{-\log f(\mathbf{Y}\,|\,\hat{\mathbf{R}}_{\mathrm{ML}}) + L(\nu(P), K)\right\}, && (8) \end{aligned}$$

where $L(\nu(P), K)$ is a penalty function depending on the number of degrees of freedom of the model $\nu(P)$ and on K, and $\hat{\mathbf{R}}_{\mathrm{ML}}$ denotes the Maximum Likelihood (ML) estimate of $\mathbf{R}$ for fixed P.

It is interesting to note that the penalty function is required to obtain a meaningful estimate of the rank of the covariance matrix of the primary signal. If we ignore the penalty function related to the rank, from the likelihood expression (5) it is easy to see that $f(\mathbf{Y}\,|\,\mathbf{R})$ is maximized for $\mathbf{R} = \mathbf{C}$. This can be always achieved if we allow a rank P large enough.[1]

The penalty function $L(\nu(P), K)$ can be determined via information theoretic or Bayesian criteria, such as the BIC, MDL or AIC presented previously. The model selection schemes proposed in [14], [18] and [5] fit into the general model (7) with different expressions for $f(\mathbf{Y} \,|\, \hat{\mathbf{R}}_{\mathrm{ML}})$ and for the penalty function $L(\nu(P), K)$.

3.1 Maximum Likelihood Estimation

We shall assume without loss of generality that $\mathbf{H}$ has full rank. From (4) we have that $\mathbf{R}$ corresponds to a rank-P matrix plus a diagonal matrix.

In [12] it is shown that, for a fixed rank P, the maximization of $f(\mathbf{Y} \,|\, \mathbf{R})$ with respect to $\mathbf{H}$ and $\mathbf{\Sigma}^2$ presents no closed form solution in general. However, while this optimization problem is non-convex, it is possible to partition the free variables in two different sets to obtain an alternating optimization scheme which outputs a quasi-ML estimate. This algorithm is briefly described next for completness.

The ML estimation problem $\max_{\mathbf{H},\mathbf{\Sigma}^2} f(\mathbf{Y} \,|\, \mathbf{R})$ can be rewritten as

$$\begin{aligned} \underset{\mathbf{H}_{\mathbf{\Sigma}},\mathbf{\Sigma}}{\text{minimize}} \quad & \log\det \mathbf{R}_{\mathbf{\Sigma}} - \log\det\left(\mathbf{\Sigma}^{-2}\right) + \mathrm{tr}\left(\mathbf{C}_{\mathbf{\Sigma}}\mathbf{R}_{\mathbf{\Sigma}}^{-1}\right), \quad (9) \\ \text{subject to} \quad & \mathbf{R}_{\mathbf{\Sigma}} = \mathbf{I}_M + \mathbf{H}_{\mathbf{\Sigma}}^H \mathbf{H}_{\mathbf{\Sigma}}, \\ & [\mathbf{\Sigma}]_{i,i} \geq 0, \end{aligned}$$

where we defined $\mathbf{C}_{\mathbf{\Sigma}} \doteq \mathbf{\Sigma}^{-1}\mathbf{C}\mathbf{\Sigma}^{-1}$ (the *whitened* sample covariance matrix) and $\mathbf{H}_{\mathbf{\Sigma}} \doteq \mathbf{H}\mathbf{\Sigma}^{-1}$.

From (9), we note that the individual minimization with respect to $\mathbf{\Sigma}$ (considering $\mathbf{H}_{\mathbf{\Sigma}}$ fixed) and with respect to $\mathbf{H}_{\mathbf{\Sigma}}$ (considering $\mathbf{\Sigma}$ fixed) can be easily written as convex problems individually, and, therefore, they can be efficiently solved.

Minimization with respect to $\mathbf{H}_{\mathbf{\Sigma}}$. For fixed $\mathbf{\Sigma}$, the optimal $\mathbf{H}_{\mathbf{\Sigma}}$ minimizing (9) is (up to a left multiplication by an arbitrary unitary matrix) given by [12, Lemma 4], that is

$$\hat{\mathbf{H}}_{\mathbf{\Sigma}} = \left(\mathrm{diag}\left(\gamma_1, \ldots, \gamma_P\right) - \mathbf{I}_P\right)^{1/2} \begin{bmatrix} \mathbf{q}_1 & \cdots & \mathbf{q}_P \end{bmatrix}^H, \quad (10)$$

with $\mathbf{C}_{\mathbf{\Sigma}} = \mathbf{Q}\mathrm{diag}\left(\gamma_1, \ldots, \gamma_M\right)\mathbf{Q}^H$ denoting the EVD of $\mathbf{C}_{\mathbf{\Sigma}}$, with $\gamma_1 \geq \cdots \geq \gamma_M$.

Minimization with respect to $\mathbf{\Sigma}$. For fixed $\mathbf{H}_{\mathbf{\Sigma}}$ the minimization problem in (9) reduces to

$$\begin{aligned} \underset{\mathbf{\Sigma}}{\text{minimize}} \quad & \mathrm{tr}\left(\mathbf{C}\mathbf{\Sigma}^{-1}\mathbf{R}_{\mathbf{\Sigma}}^{-1}\mathbf{\Sigma}^{-1}\right) - \log\det\left(\mathbf{\Sigma}^{-2}\right) \quad (11) \\ \text{subject to} \quad & [\mathbf{\Sigma}]_{i,i} \geq 0. \end{aligned}$$

Defining the vector $\boldsymbol{\alpha} = \left[[\mathbf{\Sigma}^{-1}]_{1,1}, \ldots, [\mathbf{\Sigma}^{-1}]_{M,M}\right]^T$, the trace term in (11) can be reorganized to obtain an equivalent minimization problem given by

$$\begin{aligned} \underset{\boldsymbol{\alpha}}{\text{minimize}} \quad & \boldsymbol{\alpha}^T(\mathbf{C}^T \odot \mathbf{R}_{\mathbf{\Sigma}}^{-1})\boldsymbol{\alpha} - \sum_{i=1}^{M} \log \alpha_i^2 \quad (12) \\ \text{subject to} \quad & \alpha_i \geq 0. \end{aligned}$$

Note that, given the trace term in (11), the matrix $\mathbf{C}^T \odot \mathbf{R}_{\mathbf{\Sigma}}^{-1}$ is positive semidefinite. Hence, the problem (12) is convex with respect to the parameter vector $\boldsymbol{\alpha}$ and, therefore, it can be efficiently solved using any convex optimization solver.

Algorithm 1: Iterative estimation of $\mathbf{H}_{\mathbf{\Sigma}}$ and $\mathbf{\Sigma}$ via alternating optimization.

Input: Starting point $\boldsymbol{\alpha}_{(0)}$ and $\mathbf{C}$.
Output: ML estimates of $\mathbf{H}_{\mathbf{\Sigma}}$ and $\mathbf{\Sigma}$.
Initialize: $n = 0$;
repeat
  Compute $\mathbf{\Sigma}_{(n)}^{-1} = \mathrm{diag}(\boldsymbol{\alpha}_n)$;
  Obtain $\mathbf{C}_{\mathbf{\Sigma}}^{(n+1)} = \mathbf{\Sigma}_{(n)}^{-1}\mathbf{C}\mathbf{\Sigma}_{(n)}^{-1}$ and its EVD;
  Compute $\mathbf{H}_{\mathbf{\Sigma}}^{(n+1)}$ from (10) (fixed $\mathbf{\Sigma}_{(n)}^{-1}$);
  Solve (12) to obtain $\boldsymbol{\alpha}_{(n+1)}$ (fixed $\mathbf{H}_{\mathbf{\Sigma}}^{(n+1)}$) ;
  Update $n = n + 1$;
until Convergence;

The proposed alternating minimization algorithm is summarized in Alg. 1. When the estimates of $\mathbf{\Sigma}^{-1}$ and $\mathbf{H}_{\mathbf{\Sigma}}$ are available, we can construct the quasi-ML estimator of $\mathbf{R}$ as

$$\hat{\mathbf{R}}_{\mathrm{qML}} = \hat{\mathbf{\Sigma}}(\mathbf{I}_M + \hat{\mathbf{H}}_{\mathbf{\Sigma}}^H \hat{\mathbf{H}}_{\mathbf{\Sigma}})\hat{\mathbf{\Sigma}}. \quad (13)$$

While the alternating minimization approach does not guarantee that the global maximizer of the log-likelihood is found, in the numerical experiments conducted this estimator shows good performance.

3.2 Rank Estimation

Once the quasi-ML estimate $\hat{\mathbf{R}}_{\mathrm{qML}}$ is available, and using the likelihood expression (5), the rank estimation problem in (7) can be rewritten as

$$\hat{P} = \arg\min_{P} \Big\{ -\log\det(\hat{\mathbf{R}}_{\mathrm{qML}}) + \mathrm{tr}\left(\mathbf{C}\hat{\mathbf{R}}_{\mathrm{qML}}^{-1}\right) + L(\nu(P), K)/K \Big\}. \quad (14)$$

The minimization problem in (14) can be further simplified by noting that the quasi-ML estimate $\hat{\mathbf{R}}_{\mathrm{qML}}$ is a minimizer of (12). Then, it must fulfill the KKT conditions of this problem [4]. If we define the Lagrange multipliers λ_i for $i = 1, \ldots, M$, associated to each of the constraints, we have that the Lagrangian of the cost function and its gradient are respectively given by

$$J(\boldsymbol{\alpha}) = \boldsymbol{\alpha}^T(\mathbf{C}^T \odot \mathbf{R}_{\mathbf{\Sigma}}^{-1})\boldsymbol{\alpha} - \sum_{i=1}^{M} \log \alpha_i^2 - \sum_{i=1}^{M} \lambda_i \alpha_i, \quad (15)$$

$$\frac{\partial J(\boldsymbol{\alpha})}{\partial \alpha_i} = 2\left(\boldsymbol{\alpha}^T(\mathbf{C}^T \odot \mathbf{R}_{\mathbf{\Sigma}}^{-1})\mathbf{e}_i - \frac{1}{\alpha_i} - \frac{\lambda_i}{2}\right). \quad (16)$$

[1] For example, in the extreme case $P = M$, $\mathbf{H} = \mathbf{C}^{1/2}$ and $\mathbf{\Sigma}^2 = \mathbf{0}$, obtaining $\mathbf{R} = \mathbf{H}^H\mathbf{H} + \mathbf{\Sigma}^2 = \mathbf{C}$.

Equating (16) to zero and rearranging terms we obtain

$$\hat{\boldsymbol{\alpha}}^T(\mathbf{C}^T \odot \hat{\mathbf{R}}_{\boldsymbol{\Sigma}}^{-1})(\hat{\alpha}_i \mathbf{e}_i) = 1 + \frac{1}{2}\lambda_i \alpha_i, \qquad (17)$$

for $i = 1, \ldots, M$. However, due to the complementary slackness condition, $\lambda_i \alpha_i = 0$ for all i. Using this property and adding together the equations (17) for $i = 1, \ldots, M$ we obtain

$$\hat{\boldsymbol{\alpha}}^T(\mathbf{C}^T \odot \hat{\mathbf{R}}_{\boldsymbol{\Sigma}}^{-1})\hat{\boldsymbol{\alpha}} = M. \qquad (18)$$

Therefore, we have that the quasi-ML estimate $\hat{\mathbf{R}}_{\mathrm{qML}}$ fullfils

$$\begin{aligned} \mathrm{tr}\left(\mathbf{C}\hat{\mathbf{R}}_{\mathrm{qML}}^{-1}\right) &= \mathrm{tr}\left(\mathbf{C}\hat{\boldsymbol{\Sigma}}^{-1}\hat{\mathbf{R}}_{\boldsymbol{\Sigma}}^{-1}\hat{\boldsymbol{\Sigma}}^{-1}\right) & (19) \\ &= \hat{\boldsymbol{\alpha}}^T(\mathbf{C}^T \odot \hat{\mathbf{R}}_{\boldsymbol{\Sigma}}^{-1})\hat{\boldsymbol{\alpha}} = M. & (20) \end{aligned}$$

Finally, noting that (20) does not depend on P, we may rewrite (14) as

$$\hat{P} = \arg\min_P \left\{ -\log\det(\hat{\mathbf{R}}_{\mathrm{qML}}) + L(\nu(P), K)/K \right\}. \qquad (21)$$

In our problem, the number of free-adjusted parameters $\nu(P)$ can be computed as follows: the diagonal matrix $\boldsymbol{\Sigma}^2$ presents M real parameters and the P eigenvalues and eigenvectors conforming the $\mathbf{H}$ matrix correspond to $P + P(2M - (P+1))$ real parameters. Then the total number of free real parameters is given by

$$\nu(P) = M + P(2M - P). \qquad (22)$$

4. NUMERICAL RESULTS

In this section, we will restrict our analysis to the BIC criterion based on a Bayesian approach given by $L(\nu(P), K) = \nu(P)/2 \log K$. Then, from (22) we have that

$$L(\nu(P), K) = \frac{M + P(2M - P)}{2} \log K. \qquad (23)$$

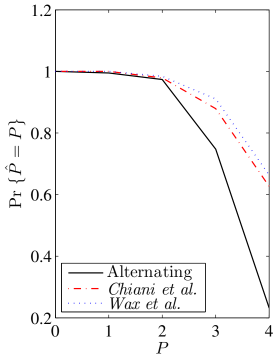
We now compare the proposed rank estimation scheme (14) with the schemes proposed by Wax et al. in [18] and Chiani et al. in [5].

The scheme by Wax et al. uses the closed form ML estimates of the channel to primary $\mathbf{H}$ and of the noise power σ^2 assuming that all the antennas present equal noise level. The estimator is given by [18]

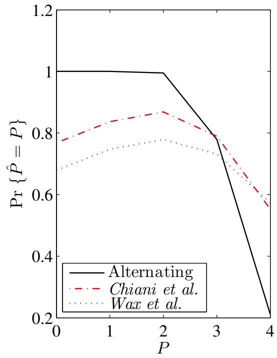
$$\begin{aligned} \hat{P}_{\mathrm{Wax}} = \arg\min_P \Bigg\{ & K(M-P)\log\left(\frac{\frac{1}{M-P}\sum_{i=P+1}^M l_i}{\prod_{i=P+1}^M l_i^{1/(M-P)}}\right) \\ & + L(P(2M-P), K)\Bigg\}, \qquad (24) \end{aligned}$$

where $l_1 \geq l_2 \geq \ldots \geq l_M$ denote the eigenvalues of $\mathbf{C}$.

The scheme proposed by Chiani et al. can be found in [5] and it is very similar to (24). The only difference is that the scheme by Chiani et al. does not use the estimate of the whole matrix $\mathbf{H}$. Instead, it estimates only the eigenvalues of $\mathbf{H}$, showing a better performance for short data records. The drawbacks of this scheme are its increased complexity and sensitivity to numerical problems as the record-length increases.



(a)



(b)

Figure 1: Probability of correct estimation versus the rank of the primary signal. (a) iid noise (b) non-iid noise.

Unless otherwise specified, the noise level at each antenna is fixed for each experiment, and for each Monte Carlo realization the entries of the channel matrix $\mathbf{H}$ are independently drawn from a complex Gaussian distribution (thus obtaining a Rayleigh fading scenario) and scaled so that the SNR is constant during the experiment:

$$\mathrm{SNR}\ (\mathrm{dB}) = 10\log_{10}\frac{\mathrm{tr}(\mathbf{H}\mathbf{H}^H)/P}{\mathrm{tr}(\boldsymbol{\Sigma}^2)/M}. \qquad (25)$$

4.1 Performance versus the number of sources

Here we compare the performance of the proposed detectors in two different scenarios with $M = 6$ antennas, $K = 32$ samples and SNR equal to 10 dB. While the first assumes that all the antennas present a noise level equal to 0 dB, in the second we consider a noise mismatch exists between different antennas. In particular we assume that each of the antennas presents a noise level equal to 0, -2, 3, 0, -3 and 2 dB, respectively.

Fig. 1 shows the probability of correct estimation versus the rank of the primary signal for the three detectors considered. We can see that for iid noises the estimators proposed by Chiani et al. and Wax et al. outperform the proposed scheme. This is intuitively satisfying since these schemes have been derived under the iid assumption. However, when a noise level mismatch exist, the Chiani et al. and Wax et al. methods suffer a strong degradation while the proposed scheme performance remains unaffected.

It is interesting to note that, both in the i.i.d and non-iid cases the probability of error increases with the rank of the signal to detect. This can be attributed to the loss of diagonal structure of the received covariance matrix when the number of signal streams grows. This structure is completely lost [12] for signal ranks $P > M - \sqrt{M}$, i.e. in our example for $P \geq 4$. This fact explains the poor performance obtained by the proposed algorithm when $P = 4$.

4.2 Performance versus the SNR

Here we compare the performance of the proposed estimator for growing values of the SNR. We consider now an scenario

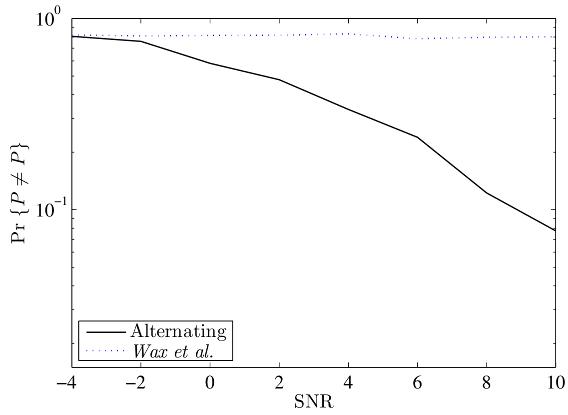


Figure 2: Probability of error versus the SNR under non-iid noises.

with $M = 6$ antennas, $K = 128$ samples and uncalibrated antennas. We use the same noise mismatch as in Fig. 1(b). The scheme proposed by Chiani et al. has not been considered in this section due to numerical precision issues arising from the fact of considering a relatively large number of samples ($K = 128$).

We assume that the rank P is randomly chosen between 0 and $M - 1$ in each Monte Carlo realization, so that Fig. 2 shows the average probability of error in the estimation process. From Fig. 2 it is apparent that the average probability of error of the proposed scheme decreases for growing values of the SNR. On the other hand, the scheme by Wax et al. presents a very poor performance due to the model mismatch when an uncalibrated receiver is considered.

5. CONCLUSIONS

Under Gaussianity assumption on both noise and data, we proposed a novel spatial rank estimation algorithm which is robust to mismatch in the noise levels at different antennas. The proposed estimator was compared to other schemes designed for calibrated receivers. While previous estimators suffer from a strong degradation in uncalibrated scenarios, the proposed scheme shows a good performance both under iid and non-iid noises.

6. ACKNOWLEDGMENTS

This work was supported by the Galician Regional Government, the Spanish Government's Ministerio de Ciencia e Innovación (MICINN) and the European Regional Development Fund (ERDF), under projects "Consolidation of Research Units" 2009/62 and 2010/85, DYNACS (TEC2010-21245-C02-02/TCM), COSIMA (TEC2010-19545-C04-03), and COMONSENS (CONSOLIDER-INGENIO 2010 CSD2008-00010).

7. REFERENCES

[1] 3GPP. LTE Release 9. *3GPP*, 2009.

[2] I. Akyildiz, W.-Y. Lee, M. Vuran, and S. Mohanty. A survey on spectrum management in cognitive radio networks. *IEEE Comm. Mag.*, 46(4):40–48, Apr. 2008.

[3] A.-J. Boonstra and A.-J. Van der Veen. Gain calibration methods for radio telescope arrays. *IEEE Trans. Signal Process.*, 51:25–38, 2003.

[4] S. Boyd and L. Vandenberghe. *Convex optimization*. Cambridge University Press, 2004.

[5] M. Chiani and M. Win. Estimating the number of signals observed by multiple sensors. In *Proc. Int. Work. Cognitive Inf. Process.*, Elba, Italy, Jun. 2010.

[6] ETSI. Digital Video Broadcasting (DVB); Frame structure channel coding and modulation for a second generation digital terrestrial television broadcasting system (DVB-T2). *ETSI EN 302 755*, 2009.

[7] IEEE Computer Society. IEEE Standard 802.11n-2009. Part 11: Wireless LAN Medium Access Control (MAC) and Physical Layer (PHY) Specifications. Amendment 5: Enhancements for Higher Throughput. 2009.

[8] IEEE Computer Society and the IEEE Microwave Theory and Techniques Society. IEEE Standard 802.16-2009. Part 16: Air Interface for Broadband Wireless Access Systems. 2009.

[9] R. López-Valcarce, G. Vazquez-Vilar, and J. Sala. Multiantenna spectrum sensing for cognitive radio: overcoming noise uncertainty. In *Proc. Int. Work. Cognitive Inf. Process.*, Elba, Italy, Jun. 2010.

[10] J. Mitola and G. Q. Maguire Jr. Cognitive radio: Making software radios more personal. *IEEE Pers. Comm.*, 6:13–18, Aug. 1999.

[11] J. M. Peha. Sharing spectrum through spectrum policy reform and cognitive radio. *Proc. IEEE*, 97(4):708–719, Apr. 2009.

[12] D. Ramirez, G. Vazquez-Vilar, R. Lopez-Valcarce, J. Via, and I. Santamaria. Detection of rank-p signals in cognitive radio networks with uncalibrated multiple antennas. *IEEE Trans. Signal Process.*, 59(8):3764 –3774, aug. 2011.

[13] D. Ramirez, G. Vazquez-Vilar, R. L.-V. J. Via, and I. Santamaria. Multiantenna detection under noise uncertainty and primary user's spatial structure. In *2011 IEEE Int. Conf. on Acoust., Speech and Signal Process.*, Prague, Czech Republic, May 2011.

[14] G. Schwarz. Estimating the dimension of a model. *Ann. Statist*, 6(2):461 – 464, 1978.

[15] A. Taherpour, M. Nasiri-Kenari, and S. Gazor. Multiple antenna spectrum sensing in cognitive radios. *IEEE Trans. Wireless Comm.*, 9(2):814–823, Feb. 2010.

[16] J. Tellado. *Multicarrier modulation with low PAR: Applications to DSL and wireless*. Kluwer Academic, Norwell, MA, 2000.

[17] P. Wang, J. Fang, N. Han, and H. Li. Multiantenna-assisted spectrum sensing for cognitive radio. *IEEE Trans. Vehicular Tech.*, 59(4):1791–1800, May 2010.

[18] M. Wax and T. Kailath. Detection of signals by information theoretic criteria. *IEEE Trans. Acoust., Speech and Signal Process.*, 33(2):387 – 392, apr 1985.

[19] R. Zhang, T. J. Lim, Y.-C. Liang, and Y. Zeng. Multi-antenna based spectrum sensing for cognitive radios: a GLRT approach. *IEEE Trans. Comm.*, 58(1):84–88, Jan. 2010.