MULTI-SENSOR BEAMSTEERING BASED ON THE ASYMPTOTIC LIKELIHOOD FOR COLORED SIGNALS

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ABSTRACT

In this work, we derive a maximum likelihood formula for beamsteering in a multi-sensor array. The novelty of the work is that the impinging signal and noises are wide sense stationary (WSS) time series with unknown power spectral densities, unlike in previous work that typically considers white signals. Our approach naturally provides a way of fusing frequency-dependent information to obtain a broadband beamformer. In order to obtain the compressed likelihood, it is necessary to find the maximum likelihood estimates of the unknown parameters. However, this problem turns out to be an ML estimation of a block-Toeplitz matrix, which does not have a closed-form solution. To overcome this problem, we derive the asymptotic likelihood, which is given in the frequency domain. Finally, some simulation results are presented to illustrate the performance of the proposed technique. In these simulations, it is shown that our approach presents the best results.

***Index Terms*—** Array processing, maximum likelihood (ML) estimation, compressed likelihood, bearing response pattern.

1. INTRODUCTION

In many applications of broadband array processing such as radar, sonar, biomedical imaging and wireless communications, it is important to obtain an array's bearing response pattern from a set of measurements [1]. In this work, we propose to use the compressed likelihood, that is, the likelihood after maximum likelihood (ML) estimation of all nuisance parameters, to steer a beam. The idea of using the compressed likelihood is not new [2, 3], however, we generalized the approach by considering broadband signals and noises with unknown spectral shape. Several techniques exist that consider signals with temporal correlation, see [4] and references therein. In particular, in [4], a narrowband Capon beamformer is obtained at each frequency and this frequency-dependent information is fused using heuristic approaches such as, arithmetic, geometric and harmonic means.

One technical problem of our approach is that it is necessary to obtain the ML estimate of a block-Toeplitz covariance matrix, which is known to be a problem with no closed-form solution [5–7]. To overcome this, and similar to our previous works in detection [8–10], we propose to use the asymptotic (frequency domain) likelihood, which naturally provides a way to fuse frequency-dependent information.

Finally, the performance of the proposed approach is illustrated by means of numerical simulations. The bearing response pattern for the ML beamsteering is compared to that of the estimator which ignores the information about the temporal structure [2, 3], showing the advantages of exploiting the temporal correlation in the estimation process. Additionally, we use the bearing response patterns to estimate the angle of arrival (AOA), and it is shown that the proposed approach outperforms well known techniques, such as MUSIC [11], by roughly 6 dB in input signal-to-noise ratio (SNR) for the broadband example we consider. This difference is reduced to roughly 2 dB when our approach is compared to one of those presented in [4].

2. PROBLEM STATEMENT

In this paper, we derive a beamsteering algorithm based on compressed likelihood for the model[1]

$$\mathbf{x}[n] = \mathbf{h}(\phi)\, d[n] + \mathbf{v}[n], \quad n = 0, \ldots, N-1, \tag{1}$$

where $\mathbf{x}[n] \in \mathbb{C}^L$ are array measurements and

$$\mathbf{h}(\phi) = \frac{1}{\sqrt{L}} \begin{bmatrix} 1 & e^{j\phi} & \cdots & e^{j(L-1)\phi} \end{bmatrix}^T,$$

[1]This model is valid for a sequence $d[n]$ which satisfies that BW/f is small enough (compared to the array length), where BW is the bandwidth of the signal $d[n]$ and f is the carrier frequency.

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is the array's steering vector; $d[n]$ is the zero-mean wide sense stationary (WSS) transmitted or radiated signal with covariance function $r_d[n] = E[d[m]d^*[m-n]]$, and $\mathbf{v}[n] \in \mathbb{C}^L$ is the independent and identically distributed (iid) noise vector whose matrix-valued covariance function is $\mathbf{R}_v[n] = E\left[\mathbf{v}[m]\mathbf{v}^H[m-n]\right] = r_v[n]\mathbf{I}$. We model $\mathbf{v}[n]$ and $d[n]$ as circular complex Gaussian stochastic processes. In (1), the transmitted signal and noise are colored, whereas in previous work which uses the compressed likelihood for beamsteering, it is assumed that they are frequency-flat [2, 3].

Let us start by constructing the data matrix

$$\mathbf{X} = \begin{bmatrix}\mathbf{x}[0] & \mathbf{x}[1] & \ldots & \mathbf{x}[N-1]\end{bmatrix} \in \mathbb{C}^{L\times N},$$

where the i-th row is an N-samples window of the time series $\{x_i[n]\}$ at the i-th sensor, and the n-th column is the n-th time sample of the vector-valued time series observed on the $L-$element array. Let us define the vector $\mathbf{z} = \text{vec}(\mathbf{X}) \in \mathbb{C}^{LN}$, which stacks the columns of $\mathbf{X}$. Hence, taking into account the WSS assumption, $\mathbf{z}$ is normally distributed with block-Toeplitz covariance matrix $\mathcal{R}_{\mathbf{x}}(\phi)$, i.e. $\mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathcal{R}_{\mathbf{x}}(\phi))$, where

$$\mathcal{R}_{\mathbf{x}}(\phi) = \begin{bmatrix} \mathbf{R}_{\mathbf{x}}[0,\phi] & \mathbf{R}_{\mathbf{x}}[-1,\phi] & \cdots & \mathbf{R}_{\mathbf{x}}[-N+1,\phi] \\ \mathbf{R}_{\mathbf{x}}[1,\phi] & \mathbf{R}_{\mathbf{x}}[0,\phi] & \cdots & \mathbf{R}_{\mathbf{x}}[-N+2,\phi] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{\mathbf{x}}[N-1,\phi] & \mathbf{R}_{\mathbf{x}}[N-2,\phi] & \cdots & \mathbf{R}_{\mathbf{x}}[0,\phi] \end{bmatrix},$$

with the matrix-valued covariance function given by

$$\mathbf{R}_{\mathbf{x}}[n,\phi] = \mathbf{h}(\phi)\, r_d[n]\mathbf{h}^H(\phi) + \mathbf{r}_v[n]\mathbf{I}. \tag{2}$$

Then, considering M independent and identically distributed (iid) realizations of vector $\mathbf{z}$, i.e. $\mathbf{z}_m = [\mathbf{x}_m^T[0], \ldots, \mathbf{x}_m^T[N-1]]^T, m = 0, \ldots, M-1$, the log-likelihood is given by

$$\log p(\mathbf{z}_0, \ldots, \mathbf{z}_{M-1}; \mathcal{R}_{\mathbf{x}}(\phi)) = -LNM\log\pi - M\log\det[\mathcal{R}_{\mathbf{x}}(\phi)] - M\text{tr}\left[\hat{\mathcal{R}}_{\mathbf{x}}\mathcal{R}_{\mathbf{x}}^{-1}(\phi)\right], \tag{3}$$

where

$$\hat{\mathcal{R}}_{\mathbf{x}} = \frac{1}{M}\sum_{m=0}^{M-1}\mathbf{z}_m\mathbf{z}_m^H.$$

Hence, to obtain a beamsteering algorithm based on compressed likelihood, we must obtain the maximum likelihood (ML) estimates of $r_d[n]$ and $r_v[n]$ for $n = 0, \ldots, N-1$, plug them into (3) and sweep out the compressed likelihood for beamsteering vector $\mathbf{h}(\phi), -\pi < \phi \leq \pi$, which is the last remaining parameter of $\log p(\mathbf{z}_0, \ldots, \mathbf{z}_{M-1}; \mathcal{R}_{\mathbf{x}}(\phi))$. This is the essence of our approach to broadband fusing.

3. BEARING RESPONSE PATTERN BASED ON THE ASYMPTOTIC LIKELIHOOD

In order to obtain the compressed likelihood, we need to find the ML estimate of a block-Toeplitz matrix, which is a problem with no closed-form solution [5–7]. This limitation may be overcome by applying the asymptotic likelihood [8–10], which is given by

$$\begin{aligned}\log p\left(\mathbf{z}_0, \ldots, \mathbf{z}_{M-1}; \mathbf{S}_{\mathbf{x}}\left(e^{j\theta}, \phi\right)\right) &= -LNM\log\pi \\ &- NM\int_{-\pi}^{\pi}\log\det\mathbf{S}_{\mathbf{x}}\left(e^{j\theta},\phi\right)\frac{d\theta}{2\pi} \\ &- NM\int_{-\pi}^{\pi}\text{tr}\left[\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right)\mathbf{S}_{\mathbf{x}}^{-1}\left(e^{j\theta},\phi\right)\right]\frac{d\theta}{2\pi},\end{aligned} \tag{4}$$

where $\mathbf{S}_{\mathbf{x}}\left(e^{j\theta},\phi\right) = \mathcal{F}(\mathbf{R}_{\mathbf{x}}[n,\phi])$ is the theoretical power spectral density (PSD) matrix, $\mathcal{F}(\cdot)$ denotes the Fourier transform and the sample PSD matrix is given by

$$\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right) = \frac{1}{M}\sum_{m=0}^{M-1}\mathbf{x}_m\left(e^{j\theta}\right)\mathbf{x}_m^H\left(e^{j\theta}\right)$$

with $\mathbf{x}_m\left(e^{j\theta}\right) = 1/\sqrt{N}\sum_{n=0}^{N-1}\mathbf{x}_m[n]\,e^{-j\theta n}$. In [8], it is proven that the asymptotic likelihood converges in the mean-square sense to the conventional (time-domain) likelihood. Now, taking into account (2), the PSD matrix is given by

$$\mathbf{S}_{\mathbf{x}}\left(e^{j\theta},\phi\right) = \mathbf{h}(\phi)\, S_d\left(e^{j\theta}\right)\mathbf{h}^H(\phi) + S_v\left(e^{j\theta}\right)\mathbf{I}, \tag{5}$$

where $S_d\left(e^{j\theta}\right) = \mathcal{F}(r_d[n])$ and $S_v\left(e^{j\theta}\right) = \mathcal{F}(r_v[n])$, and substituting it into (4), the log-likelihood becomes[2]

$$\begin{aligned}&\log p\left(\mathbf{z}_0, \ldots, \mathbf{z}_{M-1}; \mathbf{h}(\phi), S_d\left(e^{j\theta}\right), S_v\left(e^{j\theta}\right)\right) = \\ &-\int_{-\pi}^{\pi}\log\det\left[\mathbf{h}(\phi)S_d\left(e^{j\theta}\right)\mathbf{h}^H(\phi) + S_v\left(e^{j\theta}\right)\mathbf{I}\right]\frac{d\theta}{2\pi} \\ &-\int_{-\pi}^{\pi}\text{tr}\left\{\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right)\left[\mathbf{h}(\phi)S_d\left(e^{j\theta}\right)\mathbf{h}^H(\phi) + S_v\left(e^{j\theta}\right)\mathbf{I}\right]^{-1}\right\}\frac{d\theta}{2\pi}.\end{aligned}$$

Defining $\alpha\left(e^{j\theta},\phi\right) = \mathbf{h}^H(\phi)\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right)\mathbf{h}(\phi)$, which may be seen as a *frequency-dependent energy estimate in direction* ϕ, and applying the matrix inversion and determinant lemmas, we get

$$\begin{aligned}&\log p\left(\mathbf{z}_0, \ldots, \mathbf{z}_{M-1}; \mathbf{h}(\phi), S_d\left(e^{j\theta}\right), S_v\left(e^{j\theta}\right)\right) = \\ &-\int_{-\pi}^{\pi}\log\left(1 + \frac{S_d\left(e^{j\theta}\right)}{S_v\left(e^{j\theta}\right)}\right)\frac{d\theta}{2\pi} - L\int_{-\pi}^{\pi}\log S_v\left(e^{j\theta}\right)\frac{d\theta}{2\pi} \\ &-\int_{-\pi}^{\pi}\frac{1}{S_v\left(e^{j\theta}\right)}\text{tr}\left[\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right)\right]\frac{d\theta}{2\pi} \\ &+\int_{-\pi}^{\pi}\frac{S_d\left(e^{j\theta}\right)\alpha\left(e^{j\theta},\phi\right)}{S_v^2\left(e^{j\theta}\right) + S_v\left(e^{j\theta}\right)S_d\left(e^{j\theta}\right)}\frac{d\theta}{2\pi}.\end{aligned}$$

[2]For notational simplicity, constant and factor terms will be dropped.

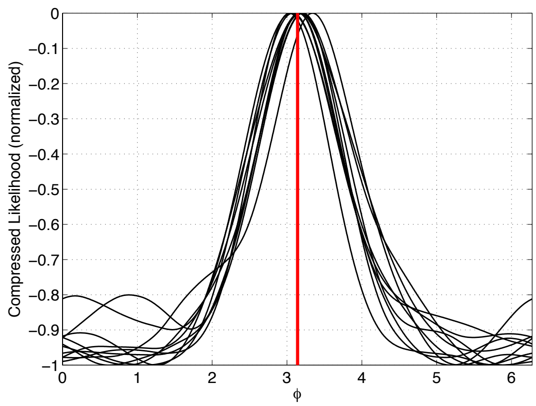

Fig. 1. Bearing response patterns based on the asymptotic likelihood.

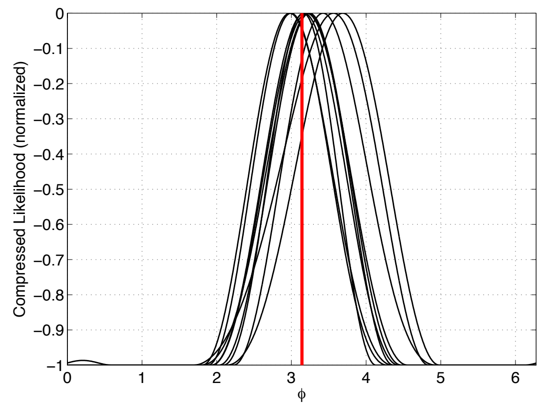

Fig. 2. Bearing response pattern based on the likelihood.

To obtain the ML estimate of the PSD of the signal, assuming for the moment $S_v\left(e^{j\theta}\right)$ known, we shall solve the following optimization problem

$$\begin{aligned} \underset{S_d(e^{j\theta})}{\text{maximize}} \quad & \log p\left(\mathbf{z}_0, \ldots, \mathbf{z}_{M-1}; \mathbf{h}(\phi), S_d\left(e^{j\theta}\right), S_v\left(e^{j\theta}\right)\right), \\ \text{subject to} \quad & S_d\left(e^{j\theta}\right) \geq 0. \end{aligned}$$

Taking into account the Karush-Kuhn-Tucker (KKT) conditions, the following ML estimate is obtained

$$\hat{S}_d\left(e^{j\theta}, \phi\right) = \left[\alpha\left(e^{j\theta}, \phi\right) - S_v\left(e^{j\theta}\right)\right]^+,$$

where $[a]^+ = \max(a, 0)$.

Now, we shall consider two different cases: 1) $\hat{S}_d\left(e^{j\theta}, \phi\right) = 0$ and 2) $\hat{S}_d\left(e^{j\theta}, \phi\right) > 0$. In the first case it is easy to show that the ML estimate of $S_v\left(e^{j\theta}\right)$ is [9]

$$\hat{S}_v\left(e^{j\theta}, \phi\right) = \frac{1}{L}\text{tr}\left[\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right)\right],$$

whereas in the second case, after some tedious algebra, the ML estimate of $S_v\left(e^{j\theta}\right)$ becomes

$$\hat{S}_v\left(e^{j\theta}, \phi\right) = \frac{1}{L-1}\left(\text{tr}\left[\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right)\right] - \alpha\left(e^{j\theta}, \phi\right)\right),$$

which may be interpreted as a *frequency-dependent estimate of the energy per spatial dimension in the noise subspace*. Although we have not imposed the non-negativity constraint on the ML estimate of $S_v\left(e^{j\theta}, \phi\right)$, it is easy to check that both estimates fulfill such restriction. Thus, in order to obtain the compressed log-likelihood in both cases ($\hat{S}_d\left(e^{j\theta}, \phi\right) > 0$ and $\hat{S}_d\left(e^{j\theta}, \phi\right) = 0$), we define

$$\beta\left(e^{j\theta}, \phi\right) = \max\left\{\alpha\left(e^{j\theta}, \phi\right), \frac{1}{L}\text{tr}\left[\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right)\right]\right\},$$

which is the maximum of the energy in direction ϕ and the average energy per dimension, at frequency θ. Finally, taking into account the ML estimates, the compressed log-likelihood becomes

$$\log p\left(\mathbf{z}_0, \ldots, \mathbf{z}_{M-1}; \mathbf{h}(\phi)\right) = -\int_{-\pi}^{\pi} \log\beta\left(e^{j\theta}, \phi\right)\frac{d\theta}{2\pi}$$
$$-(L-1)\int_{-\pi}^{\pi}\log\left[\frac{1}{L-1}\left(\text{tr}\left[\hat{\mathbf{S}}_{\mathbf{x}}\left(e^{j\theta}\right)\right] - \beta\left(e^{j\theta}, \phi\right)\right)\right]\frac{d\theta}{2\pi}. \quad (6)$$

The broadband bearing response pattern of the ML beamsteering algorithm is obtained by sweeping out the compressed log-likelihood given by (6) for steering angles $-\pi < \phi \leq \pi$. As can be seen in (6), the proposed approach naturally provides the right way to fuse the array information at different frequencies.

4. NUMERICAL RESULTS

In this section we present some simulations to illustrate the performance of the proposed technique (Eq. (6)) and compare it to a compressed likelihood which does not take into account the time-domain information [3]. For the simulation we have used the following parameters: the number of sensors of the antenna array is $L = 3$, there are $M = 3$ realizations of length $N = 128$ for the estimation process and the input signal-to-noise ratio is defined as $\text{SNR} = 10\log\left(\frac{r_d[0]}{L\sigma^2}\right) = -10$ dB, where $L\sigma^2$ is the total variance of the noise vector. Furthermore, we have considered that the signal and noises are moving average (MA) stochastic processes of order 4. In Fig. 1 we have plotted ten different realizations of the bearing response patterns given by (6) and, in Fig. 2, ten realizations of the bearing response pattern of the algorithm in [3]. Additionally, in red (vertical) line, the true angle of arrival (AOA) is plotted. It is important to note that the bearing response patterns have been normalized between 0 and -1, to better illustrate the differences of both approaches. As can be seen, the proposed algorithm presents the best results.

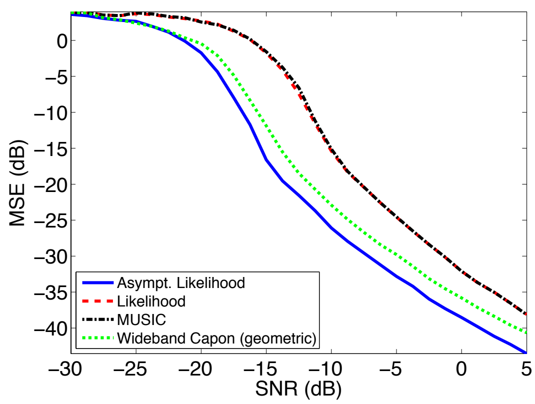

Fig. 3. MSE in the estimated AOA.

To obtain results with some statistical significance, which better illustrate the differences between both approaches, we propose to use the bearing response pattern to estimate AOA. The estimate is obtained using a line search over a grid of 1024 points. Fig. 3 shows the mean square error (MSE) of both approaches, with the MSE defined as follows MSE (dB) $= 20 \log \left| \hat{\phi} - \phi \right|$, where $\hat{\phi}$ is the estimated AOA and ϕ is the true AOA. Additionally, we have also included the MUSIC [11] approach[3] and the wideband Capon beamforming based on the geometric mean [4]. It can be seen that the broadband asymptotic likelihood-based estimator outperforms the likelihood-based and MUSIC-based estimators, which obtain almost the same MSE. The reason for this better performance is that the asymptotic likelihood-based estimator fully exploits the space-time structure of the problem. Additonally, the proposed algorithm also outperforms the wideband Capon beamformer. Over a wide range of input SNRs the improvement is on the order of 6 dB in input SNR compared to the algorithms which ignore the temporal structure and around 2 dB compared to the wideband Capon beamformer.

5. CONCLUSIONS

In this paper we have derived a broadband beamsteering algorithm based on compressed likelihood, i.e. the likelihood after obtaining the maximum likelihood (ML) estimates of all nuisance parameters. Our particular model is a colored signal impinging on an array which is distorted with a frequency-selective but spatially white noise. The problem of obtaining the ML estimates of the unknown parameters amounts to finding the ML estimate of a block-Toeplitz matrix. This is a non-convex problem with no closed-form solution, which is overcome by resorting to the asymptotic likelihood. Interestingly, our approach provides a natural way of fusing the frequency-dependent bearing response patterns, contrary to previous work, which uses heuristic approaches. Finally, simulations results illustrate the good performance of the proposed technique.

[3]For the simulations, only the MUSIC approach which ignores the time structure is considered.

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