

MULTIPLE-CHANNEL DETECTION OF A GAUSSIAN TIME SERIES OVER FREQUENCY-FLAT CHANNELS

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ABSTRACT

This work addresses the problem of deciding whether a set of realizations of a vector-valued time series with unknown temporal correlation are spatially correlated or not. Specifically, the spatial correlation is induced by a colored source over a frequency-flat single-input multiple-output (SIMO) channel distorted by independent and identically distributed noises with temporal correlation. The generalized likelihood ratio test (GLRT) for this detection problem does not have a closed-form expression and we have to resort to numerical optimization techniques. In particular, we apply the successive convex approximations approach which relies on solving a series of convex problems that approximate the original (non-convex) one. The proposed solution resembles a power method for obtaining the dominant eigenvector of a matrix, which changes over iterations. Finally, the performance of the proposed detector is illustrated by means of computer simulations showing a great improvement over previously proposed detectors that do not fully exploit the temporal structure of the source.

Index Terms— Multiple-channel detection, generalized likelihood ratio test (GLRT), maximum likelihood (ML) estimation, convex optimization, successive convex approximations.

1. INTRODUCTION

The multiple-channel signal detection problem appears in many applications, such as sensor networks [1], radar detection with multiple antennas [2] or cognitive radio [3]. In [4], this problem has been addressed for vector-valued random variables, where the authors proposed a new measure called the generalized coherence (GC). The generalized likelihood ratio test (GLRT) for testing an unstructured covariance matrix against a diagonal one has been derived, under the Gaussian assumption, in [5]. Eventually, the GLRT in [5, 6] coincides with the GC. The results in [4, 5] were extended in [7] to the case of vector-valued time series with unknown temporal structure. On the other hand, several authors have considered the case of more structured signals. For instance, in [8], the GLRT for the detection of a vector-valued random variable with rank-one covariance matrix in independent and identically distributed (iid) noises has been derived. In [9], the case of non-iid noises has been considered, where an approximated GLRT is derived for an asymptotically low signal-to-noise ratio (SNR). Moreover, by applying the asymptotic likelihood (in the frequency domain), these results have been extended to vector-valued time series in [10].

In this work, we consider the detection of a temporally correlated signal (with arbitrary *unknown* spectral shape) which under-

goes propagation through a *frequency-flat* single-input multiple output (SIMO) channel and is distorted by independent and identically distributed noises. This might be the case of detecting a typical communications signal without performing synchronization, i.e. the detection is made just after the analog-to-digital (ADC) conversion. Another example for this model is a communications system transmitting a signal that has been linearly precoded using an unknown filter. In this scenario, even if the synchronization and sampling at the symbol rate are performed, we will obtain a temporally correlated signal over a frequency-flat channel.

To solve this detection problem, we propose to apply the GLRT. Following the lines of our previous work in [10], the maximum likelihood (ML) estimation of block-Toeplitz matrices is avoided by applying the asymptotic likelihood (in the frequency domain). Even with this approach, the ML estimation of the SIMO channel results in a complicated non-convex problem with no closed-form solution. Hence, we have to resort to numerical optimization techniques. In particular, we apply the successive convex approximations approach (or condensation method) [11, 12]. The resulting algorithm for the ML estimate of the SIMO channel resembles a power method for obtaining the dominant eigenvector of a matrix, which changes over iterations. Finally, the performance of the proposed detector is illustrated by means of computer simulations showing a great improvement over previously proposed detectors.

2. PROBLEM FORMULATION

In this paper, we address the following detection problem

$$\begin{aligned} \mathcal{H}_1 : \mathbf{x}[n] &= \mathbf{h}s[n] + \mathbf{v}[n], & n = 0, \dots, N-1, \\ \mathcal{H}_0 : \mathbf{x}[n] &= \mathbf{v}[n], & n = 0, \dots, N-1, \end{aligned} \quad (1)$$

where $\mathbf{x}[n] \in \mathbb{C}^L$ is a vector of measurements, $\mathbf{h} \in \mathbb{C}^L$ is the *unknown* frequency-flat SIMO channel,¹ $s[n]$ is the zero-mean wide sense stationary (WSS) time series whose *unknown* covariance function is $r_s[n] = E[s[m]s^*[m-n]]$ and $\mathbf{v}[n] \in \mathbb{C}^L$ is the independent and identically distributed (iid) noise vector whose *unknown* matrix-valued covariance function is $\mathbf{R}_v[n] = E[\mathbf{v}[m]\mathbf{v}^H[m-n]] = r_v[n]\mathbf{I}$. We assume that the signal and noise are complex circular Gaussian distributed. Unlike other models, where the signal, channel and noise are all either frequency-flat [4–6, 8] or frequency-selective [7, 10], we consider time-colored signals and noises but frequency-flat channels.

¹Without loss of generality, we assume $\|\mathbf{h}\|^2 = 1$, since any scaling factor can be absorbed by $s[n]$.

In order to proceed, let us construct the data matrix \mathbf{X}

$$\mathbf{X} = [\mathbf{x}[0] \quad \mathbf{x}[1] \quad \dots \quad \mathbf{x}[N-1]] \in \mathbb{C}^{L \times N},$$

where the i -th row contains N samples of the time series $\{x_i[n]\}$ at the i -th sensor, and the n -th column is the n -th time sample of the vector-valued time series. The vector $\mathbf{z} = \text{vec}(\mathbf{X})$ stacks the columns of \mathbf{X} , and taking into account the WSS assumption, its block-Toeplitz covariance matrix $\mathcal{R} \in \mathbb{C}^{LN \times LN}$ is given by

$$\mathcal{R} = \begin{bmatrix} \mathbf{R}[0] & \mathbf{R}[-1] & \dots & \mathbf{R}[-N+1] \\ \mathbf{R}[1] & \mathbf{R}[0] & \dots & \mathbf{R}[-N+2] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}[N-1] & \mathbf{R}[N-2] & \dots & \mathbf{R}[0] \end{bmatrix},$$

where $\mathbf{R}[n] = E[\mathbf{x}[m]\mathbf{x}^H[m-n]]$ is a matrix-valued covariance function. Therefore, the test in (1) may be rewritten as

$$\begin{aligned} \mathcal{H}_1 : \mathbf{z} &\sim \mathcal{CN}(\mathbf{0}_{LN}, \mathcal{R}_1), \\ \mathcal{H}_0 : \mathbf{z} &\sim \mathcal{CN}(\mathbf{0}_{LN}, \mathcal{R}_0). \end{aligned} \quad (2)$$

That is, we are testing two different block-Toeplitz matrices where each block has a different structure under each hypothesis. Under \mathcal{H}_0 each block is diagonal, whereas, under \mathcal{H}_1 , it is given by $\mathbf{R}[n] = \mathbf{h}r_s[n]\mathbf{h}^H + r_v[n]\mathbf{I}$.

To solve the hypothesis test in (2), we propose to use the generalized likelihood ratio test (GLRT). Consequently, we must find the ML estimates of the unknown parameters under both hypotheses. The ML estimation of Toeplitz matrices is a complicated problem with no closed-form solution [13, 14]. Additionally, the rank-one structure of the signal covariance matrix complicates the problem even more. To overcome the Toeplitz structure problem, we propose to use the asymptotic log-likelihood [10], which converges in the mean square sense to the *conventional* (time-domain) log-likelihood.

Let us consider M iid realizations of \mathbf{z} . Dropping constant terms for notational simplicity, the asymptotic log-likelihood [10] is given by

$$\begin{aligned} \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{S}(e^{j\theta})) &= - \int_{-\pi}^{\pi} \log \det \mathbf{S}(e^{j\theta}) \frac{d\theta}{2\pi} \\ &\quad - \int_{-\pi}^{\pi} \text{tr} \left[\hat{\mathbf{S}}(e^{j\theta}) \mathbf{S}^{-1}(e^{j\theta}) \right] \frac{d\theta}{2\pi}, \end{aligned} \quad (3)$$

where $\mathbf{S}(e^{j\theta})$ is the power spectral density (PSD) matrix, the sample PSD matrix is $\hat{\mathbf{S}}(e^{j\theta}) = \frac{1}{M} \sum_{i=0}^{M-1} \mathbf{x}_i(e^{j\theta}) \mathbf{x}_i^H(e^{j\theta})$ with $\mathbf{x}_i(e^{j\theta}) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \mathbf{x}_i[n] e^{-j\theta n}$. Using (3), the test in (2) asymptotically ($N \rightarrow \infty$) becomes

$$\begin{aligned} \mathcal{H}_1 : \mathbf{x}(e^{j\theta}) &\sim \mathcal{CN}(\mathbf{0}, \mathbf{S}_1(e^{j\theta})), \\ \mathcal{H}_0 : \mathbf{x}(e^{j\theta}) &\sim \mathcal{CN}(\mathbf{0}, \mathbf{S}_0(e^{j\theta})), \end{aligned} \quad (4)$$

where $\mathbf{S}_1(e^{j\theta}) = \mathbf{h}S_s(e^{j\theta})\mathbf{h}^H + S_v(e^{j\theta})\mathbf{I}$ denotes the PSD matrix under \mathcal{H}_1 and $\mathbf{S}_0(e^{j\theta}) = S_v(e^{j\theta})\mathbf{I}$ is the PSD matrix under \mathcal{H}_0 . $S_s(e^{j\theta})$ and $S_v(e^{j\theta})$ are, respectively, the Fourier transforms of $r_s[n]$ and $r_v[n]$.

3. DERIVATION OF THE GLRT

The GLRT for $\mathcal{H}_0 : \mathbf{S}_0(e^{j\theta}) = S_v(e^{j\theta})\mathbf{I}$, vs. $\mathcal{H}_1 : \mathbf{S}_1(e^{j\theta}) = S_s(e^{j\theta})\mathbf{h}\mathbf{h}^H + S_v(e^{j\theta})\mathbf{I}$, is based on the generalized likelihood

ratio \mathcal{L} [15]

$$\begin{aligned} \mathcal{L} &= \max_{S_v(e^{j\theta})} \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; S_v(e^{j\theta})) - \\ &\quad \max_{\mathbf{h}, S_s(e^{j\theta}), S_v(e^{j\theta})} \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h}, S_s(e^{j\theta}), S_v(e^{j\theta})). \end{aligned} \quad (5)$$

It is easy to show that the ML estimate of $S_v(e^{j\theta})$ under \mathcal{H}_0 is given by

$$\hat{S}_v(e^{j\theta}) = \frac{1}{L} \text{tr}(\hat{\mathbf{S}}(e^{j\theta})). \quad (6)$$

On the other hand, the ML estimates under \mathcal{H}_1 are more involved to find. Let us start by defining $\alpha(e^{j\theta}) = \mathbf{h}^H \hat{\mathbf{S}}(e^{j\theta}) \mathbf{h}$, which may be seen as a frequency-dependent estimate of the *total energy in the signal subspace*. Hence, applying the matrix inversion and determinant lemmas to $\mathbf{S}_1(e^{j\theta})$, the log-likelihood under \mathcal{H}_1 is given by

$$\begin{aligned} \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h}, S_s(e^{j\theta}), S_v(e^{j\theta})) &= - \int_{-\pi}^{\pi} \log \left(1 + \frac{S_s(e^{j\theta})}{S_v(e^{j\theta})} \right) \frac{d\theta}{2\pi} - L \int_{-\pi}^{\pi} \log S_v(e^{j\theta}) \frac{d\theta}{2\pi} \\ &\quad - \int_{-\pi}^{\pi} \frac{1}{S_v(e^{j\theta})} \text{tr}[\hat{\mathbf{S}}(e^{j\theta})] \frac{d\theta}{2\pi} \\ &\quad + \int_{-\pi}^{\pi} \frac{S_s(e^{j\theta}) \alpha(e^{j\theta})}{S_v^2(e^{j\theta}) + S_v(e^{j\theta}) S_s(e^{j\theta})} \frac{d\theta}{2\pi}. \end{aligned}$$

To obtain the ML estimate of $S_s(e^{j\theta})$, assuming for the moment $S_v(e^{j\theta})$ known, we may solve the following optimization problem

$$\begin{aligned} &\text{maximize}_{S_s(e^{j\theta})} \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h}, S_s(e^{j\theta}), S_v(e^{j\theta})), \\ &\text{subject to } S_s(e^{j\theta}) \geq 0, \end{aligned}$$

whose solution is given by

$$\hat{S}_s(e^{j\theta}) = \left[\alpha(e^{j\theta}) - S_v(e^{j\theta}) \right]^+,$$

where $[a]^+ = \max(a, 0)$.

Now, we shall consider two different cases: (i) $\hat{S}_s(e^{j\theta}) = 0$ and (ii) $\hat{S}_s(e^{j\theta}) > 0$. The first one reduces to \mathcal{H}_0 and, therefore, the ML estimate of $\hat{S}_v(e^{j\theta})$ is given by (6). In the second case, it is straightforward to show that the ML estimate of $S_v(e^{j\theta})$ is

$$\hat{S}_v(e^{j\theta}) = \frac{1}{L-1} \left(\text{tr}[\hat{\mathbf{S}}(e^{j\theta})] - \alpha(e^{j\theta}) \right),$$

which can be seen as an estimate of the *normalized energy (per dimension) in the noise subspace*, at frequency θ .

Let us define $\beta(e^{j\theta}) = \max(\alpha(e^{j\theta}), \text{tr}[\hat{\mathbf{S}}(e^{j\theta})]/L)$, which is the maximum of the energy in the signal subspace and the *average energy per dimension*. Thus, substituting the ML estimates of $\hat{S}_s(e^{j\theta})$ and $\hat{S}_v(e^{j\theta})$, the compressed log-likelihood becomes

$$\begin{aligned} \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h}) &= - \int_{-\pi}^{\pi} \log \beta(e^{j\theta}) \frac{d\theta}{2\pi} \\ &\quad - (L-1) \int_{-\pi}^{\pi} \log \left[\frac{1}{L-1} \left(\text{tr}[\hat{\mathbf{S}}(e^{j\theta})] - \beta(e^{j\theta}) \right) \right] \frac{d\theta}{2\pi}. \end{aligned} \quad (7)$$

The ML estimate of \mathbf{h} is given by the solution of the following optimization problem

$$\begin{aligned} & \underset{\mathbf{h}, \beta(e^{j\theta}), \alpha(e^{j\theta})}{\text{maximize}} && \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h}) \\ & \text{subject to} && \beta(e^{j\theta}) = \max\left(\alpha(e^{j\theta}), \frac{1}{L} \text{tr}\left[\hat{\mathbf{S}}(e^{j\theta})\right]\right), \\ & && \alpha(e^{j\theta}) = \mathbf{h}^H \hat{\mathbf{S}}(e^{j\theta}) \mathbf{h}, \\ & && \mathbf{h}^H \mathbf{h} = 1. \end{aligned} \quad (8)$$

This is very difficult to solve due to the convexity² of $\log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h})$ in $\beta(e^{j\theta})$ and the non-convexity of the constraints. To solve it we propose to use the successive convex approximations approach (or condensation method) [11, 12]. This method relies on solving a series of convex problems, in which the non-convex problem is replaced by a convex approximation. In [12] it is proven that, when the approximation satisfies some conditions, the condensation method converges to a point satisfying the Karush-Kuhn-Tucker (KKT) conditions of the original problem. Nevertheless, since the *original* optimization problem is not convex, this solution is not guaranteed to be the global maximum of the objective function.

We shall start by the convex approximation of (7) and we take it to be the first order Taylor's series expansion³ around a given point $\mathbf{h}^{(i)}$, i.e.,

$$\log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h}) \approx a + \left[\int_{-\pi}^{\pi} \mathbf{b}^H(e^{j\theta}) \frac{d\theta}{2\pi} \right] (\mathbf{h} - \mathbf{h}^{(i)}),$$

where $a = \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h}^{(i)})$,

$$\begin{aligned} \mathbf{b}^H(e^{j\theta}) &= \left. \frac{\partial \log p(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h})}{\partial \mathbf{h}} \right|_{\mathbf{h}=\mathbf{h}^{(i)}} \\ &= b(e^{j\theta}) \mathbf{h}^{(i)H} \hat{\mathbf{S}}(e^{j\theta}), \end{aligned}$$

with $b(e^{j\theta})$ given by

$$b(e^{j\theta}) = \begin{cases} \frac{L-1}{\text{tr}[\hat{\mathbf{S}}(e^{j\theta})] - \alpha^{(i)}(e^{j\theta})} - \frac{1}{\alpha^{(i)}(e^{j\theta})}, & \text{if } \theta \in \Theta_+^{(i)}, \\ 0, & \text{if } \theta \in \Theta_-^{(i)}, \end{cases}$$

$\alpha^{(i)}(e^{j\theta}) = \mathbf{h}^{(i)H} \hat{\mathbf{S}}(e^{j\theta}) \mathbf{h}^{(i)}$ is the estimated energy in the signal subspace at the i -th iteration, and $\Theta_+^{(i)}$ and $\Theta_-^{(i)}$ are the following sets of frequencies

$$\begin{aligned} \Theta_+^{(i)} &= \left\{ \theta \mid \alpha^{(i)}(e^{j\theta}) \geq \frac{1}{L} \text{tr}[\hat{\mathbf{S}}(e^{j\theta})] \right\}, \\ \Theta_-^{(i)} &= \left\{ \theta \mid \alpha^{(i)}(e^{j\theta}) < \frac{1}{L} \text{tr}[\hat{\mathbf{S}}(e^{j\theta})] \right\}. \end{aligned}$$

Thus, at each iteration, the optimization problem is given by

$$\begin{aligned} & \underset{\mathbf{h}}{\text{maximize}} && \mathbf{h}^{(i)H} \hat{\mathbf{R}}_w \mathbf{h}, \\ & \text{subject to} && \mathbf{h}^H \mathbf{h} = 1, \end{aligned}$$

²Remember that maximizing a convex function is a non-convex problem.

³It is easy to show that this approximation satisfies the necessary conditions of [12] and, therefore, it converges to a point satisfying the KKT conditions.

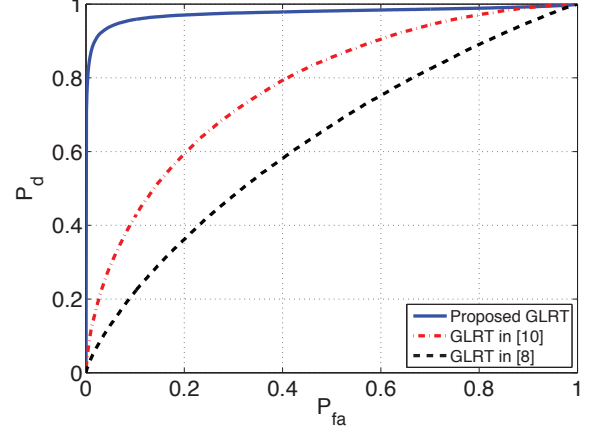


Fig. 1: ROC curve for $L = 3$, $M = 3$, $N = 128$ and $\text{SNR} = -13$ dB.

where

$$\hat{\mathbf{R}}_w = \int_{-\pi}^{\pi} b(e^{j\theta}) \hat{\mathbf{S}}(e^{j\theta}) \frac{d\theta}{2\pi},$$

and its solution is given by

$$\mathbf{h}^{(i+1)} = \begin{cases} \frac{\hat{\mathbf{R}}_w \mathbf{h}^{(i)}}{\|\hat{\mathbf{R}}_w \mathbf{h}^{(i)}\|}, & \text{if } \|\hat{\mathbf{R}}_w \mathbf{h}^{(i)}\| \neq 0, \\ \text{any unit-norm vector}, & \text{otherwise.} \end{cases}$$

This solution may be seen as one step of a power method for obtaining the dominant eigenvector of $\hat{\mathbf{R}}_w$. Due to the iterative nature of the propose approach, this estimate must be plugged into the approximated cost function and the procedure is repeated until convergence. Essentially, the proposed solution is a power method in which the matrix varies over iterations. To summarize, we have obtained the ML estimates of the unknown parameters under both hypotheses, and to obtain the log-GLRT, the ML estimates must be plugged into (5).

One final comment is in order. Although it seems that the power method is an alternative to alleviate the computational complexity of an eigenvalue extraction, it naturally results from the Taylor's approximation. Nevertheless, using the Taylor's approximation in $\alpha_i(e^{j\theta})$ instead of $\mathbf{h}^{(i)}$, we will obtain an algorithm in which the dominant eigenvector of $\hat{\mathbf{R}}_w$ is the solution at each iteration.

4. NUMERICAL RESULTS

In this section we evaluate the performance of the proposed detector by means of numerical simulations. In particular, we obtain the receiver operating characteristic (ROC) curve of the following detectors:

- The proposed detector based on the condensation method. The initial point of the algorithm is given by the dominant eigenvector of $\hat{\mathbf{R}}[0]$.
- The asymptotic GLRT which assumes that the channel is also frequency selective [10].
- The GLRT which assumes that the signal and noise are temporally uncorrelated [8].

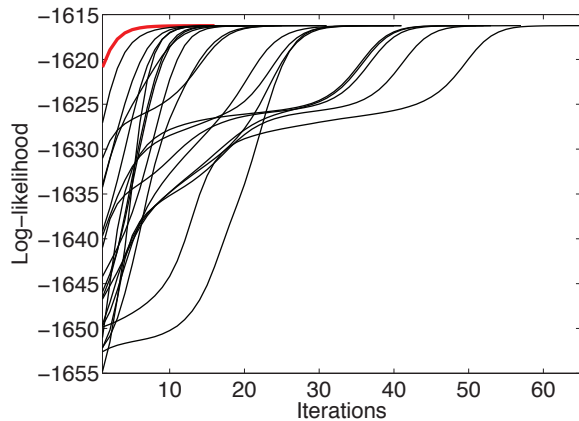


Fig. 2: Convergence curve of the condensation method for 20 random initializations.

Notice that only the proposed detector matches the space-time structure of the model, that is, only the proposed detector is actually designed for a colored source transmitted over a flat-fading channel.

Figure 1 shows the results of the experiment with the following parameters: $L = 3$ sensors, $M = 3$ realizations of length $N = 128$ and $\text{SNR} = 10 \log_{10} \frac{E_s}{L\sigma^2} = -13$ dB, where E_s is the energy of $s[n]$. The signal $s[n]$ is a moving average (MA) process of order $q_s = 19$, and the noises at each antenna are also MA process of order $q_v = 19$ with energy σ^2 . We must note that proposed detector presents much better performance than that of the detectors in [8, 10]. This may be explained by the fact that only the proposed GLRT is actually designed to exploit the assumed space-time structure of the model. Additionally, for this high frequency-selective scenario (high q_s and q_v), it is important to notice that the detector of [10] outperforms that of [8].

Figure 2 illustrates the convergence of the successive convex approximations approach. In particular, it shows the log-likelihood of 20 different initializations of $\mathbf{h}^{(0)}$, where one of them (in red thick line) is the dominant normalized eigenvector of $\hat{\mathbf{R}}[0]$. It can be seen in the figure that the algorithm converges to the same solution regardless of the initial point. Finally, we also observe in Fig. 2 that the proposed initialization notably speeds up the convergence of the successive convex approximation technique.

5. CONCLUSIONS

We have presented a new detector for a temporally correlated signal which undergoes propagation through a frequency-flat single-input multiple-output (SIMO) channel in independent and identically distributed noises with unknown temporal correlation. To solve this hypothesis testing problem, we used the generalized likelihood ratio test (GLRT). However, the ML estimation of the SIMO channel results in a complicated non-convex optimization problem with no closed-form solution. To overcome this limitation, the successive convex approximations approach (or condensation method) has been applied to obtain the ML estimate the SIMO channel. This solution resembles a power method for obtaining the dominant eigenvector of a matrix, which is obtained by integrating a weighted version of the PSD matrix. Finally, by fully exploiting the spatio-temporal structure of this problem, the proposed detector outperforms previously

proposed approaches that do not exploit all the available space-time structure.

6. ACKNOWLEDGMENTS

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