# Multiantenna spectrum sensing: The case of wideband rank-one primary signals

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***Abstract*—One of the key problems in cognitive radio (CR) is the detection of primary activity in order to determine which parts of the spectrum are available for opportunistic access. In this work, we present a new multiantenna detector which fully exploits the spatial and temporal structure of the signals. In particular, we derive the generalized likelihood ratio test (GLRT) for the problem of detecting a wideband rank-one signal under spatially uncorrelated noise with equal or different power spectral densities. In order to simplify the maximum likelihood (ML) estimation of the unknown parameters, we use the asymptotic likelihood in the frequency domain. Interestingly, for noises with different distributions and under a low SNR approximation, the GLRT is obtained as a function of the largest eigenvalue of the spectral coherence matrix. Finally, the performance of the proposed detectors is evaluated by means of numerical simulations, showing important advantages over previously proposed approaches.**

## I. Introduction

In the last years, the cognitive radio (CR) paradigm has emerged as a key technology to improve spectrum usage [1]. The basic idea behind CR is the opportunistic access of some users (secondary users) to the wireless channel when the licensed (primary) users are not transmitting. Therefore, any CR system necessarily relies on a spectrum sensing device for determining which parts of the spectrum band are available (spectrum holes). Even when a spectrum hole is found and exploited, secondary users must periodically check whether it has been reclaimed by the primary network, in which case the spectrum hole must be quickly vacated.

Detection of primary users in CR is a challenging problem because fading and shadowing may result in very weak received primary signals. This means that the spectrum monitor must be able to operate in very low SNR environments, preventing synchronization to and/or decoding of these signals, even if the modulation format and parameters of primary transmitters were known. Several detectors have been proposed for CR applications (see [2] and references therein). Perhaps the most popular (and computationally cheapest) one is the energy detector (ED), which does not require any *a priori* information about the primary system and does not need synchronization. The main drawback of the ED resides in its sensitivity to uncertainties in the background noise power, which may result in undetectable primary signals if the SNR is below certain level, *even as the observation time goes to infinity* [3]. Alternative approaches to the ED exploit some features of primary signals, such as cyclostationarity or the presence of pilots. However, these methods are sensitive to synchronization errors [4], unavoidable in low SNR conditions.

Another way to improve the detection performance of spectrum monitors consists in using multiple antennas. Intuitively, the presence of any primary signal should result in spatial correlation in the observations; a feature that can be exploited for detection since the noise processes at different antennas can be safely assumed statistically independent. This idea has been used in [5], [6] to derive the generalized likelihood ratio test (GLRT) for the problem of detecting spatially correlated signals without temporal structure. In [7] this detector was extended to consider time series with (unknown) temporal structure, showing a great improvement of the detector's performance. Nevertheless, in spite of its robustness, the performance of the detector in [7] could be improved by taking into account that, for single antenna transmitters,[1] the spatial correlation matrix at the CR node should be a rank-one matrix plus a diagonal term. This fact has been used in [8], [9] to derive the GLRT for a rank-one source without temporal structure in spatially uncorrelated white noises with equal or different variances, respectively.

In this work we propose an extension of the works in [8], [9] to consider (rank-one) signals and noises with arbitrary and unknown power spectral densities (psd), i.e., vector-valued time series with temporal correlation instead of vector-valued random variables. Concretely, we derive the asymptotic GLRT for spatially uncorrelated noises with equal or different unknown power spectral densities. The proposed GLRT is based on the asymptotic likelihood, which can be seen as an extension for multivariate processes of the Whittle's likelihood [10], and converges (in the mean square sense) to the true likelihood. Finally, numerical results are presented to show the improvement due to the exploitation of the temporal and spatial structures.

## II. Problem formulation

We consider the problem of detecting the presence of a primary user in a cognitive radio (CR) node equipped with $L$ antennas, without any prior knowledge about the primary transmission (beyond stationarity), the wireless channel, or the noise processes (beyond spatial independence and stationarity). In particular, we formulate the problem as a test for the covariance structure of the wide sense stationary (WSS) vector-valued time series $\{\mathbf{x}[n],\ n = 0, \pm 1, \ldots\}$, where $\mathbf{x}[n] = [x_1[n], \ldots, x_L[n]]^T$ is a vector of measurements at time $n$, or equivalently, $\{x_i[n]\}$ is the time series at the $i$-th antenna. The detection problem is given by

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{x}[n] = (\mathbf{h} * s)[n] + \mathbf{v}[n], \quad & n = 0, \ldots, N-1, \\ \mathcal{H}_0 &: \mathbf{x}[n] = \mathbf{v}[n], & n = 0, \ldots, N-1, \end{aligned}$$

where $s[n]$ is the primary signal, $\mathbf{h}[n] = [h_1[n], \ldots, h_L[n]]^T$ is the frequency-selective single-input multiple-output (SIMO) channel between the primary user and the CR node, $*$ denotes the convolution operator; and $\mathbf{v}[n] = [v_1[n], \ldots, v_L[n]]^T$ is the additive noise vector, which is assumed to be zero-mean circular complex Gaussian and spatially uncorrelated, i.e., $E\{v_i[n]v_k^*[m]\} = 0$ for $i \neq k$ and $\forall n, m$. Finally, we must point out that no assumptions are made neither on the *temporal* correlation of the primary signal nor on the noise processes.

[1]The case of multiantenna primary users is an interesting research line and will be considered in future work.

Let us start by defining the data matrix

$$\mathbf{X} = \begin{bmatrix}\mathbf{x}[0] & \mathbf{x}[1] & \dots & \mathbf{x}[N-1]\end{bmatrix},$$

where the $i$-th row contains $N$-samples of the $i$-th time series $\{x_i[n]\}$, and the $n$-th column is the $n$-th sample of the vector-valued time series. The vector $\mathbf{z} = \text{vec}(\mathbf{X})$ stacks the columns of $\mathbf{X}$, and taking into account the stationarity assumption, its block-Toeplitz covariance matrix is

$$\mathbf{R} = E\left[\mathbf{z}\mathbf{z}^H\right] = \begin{bmatrix} \mathbf{R}[0] & \mathbf{R}[-1] & \cdots & \mathbf{R}[-N+1] \\ \mathbf{R}[1] & \mathbf{R}[0] & \cdots & \mathbf{R}[-N+2] \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}[N-1] & \mathbf{R}[N-2] & \cdots & \mathbf{R}[0] \end{bmatrix},$$

where $\mathbf{R}[m] = E\left[\mathbf{x}[n]\mathbf{x}^H[n-m]\right]$ is a matrix-valued correlation function.

To proceed, we need the distribution of $\{\mathbf{x}[n]\}$ under $\mathcal{H}_1$. We take it to be zero-mean, circular complex Gaussian. In addition to resulting in tractable models and useful detectors, this assumption is reasonable if the primary network employs orthogonal frequency division multiplexing (OFDM) as modulation format. Thus, the hypothesis testing problem becomes

$$\mathcal{H}_1 : \mathbf{z} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_1\right),$$
$$\mathcal{H}_0 : \mathbf{z} \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{R}_0\right),$$

where $\mathcal{CN}\left(\mathbf{0}, \mathbf{R}_i\right)$ denotes the circular complex Gaussian distribution with zero mean and covariance matrix $\mathbf{R}_i$.

## III. The asymptotic likelihood

Unlike our past work presented in [7], $\mathbf{R}_1$ is now a structured matrix due to the presence of a rank-one primary source, which complicates the ML estimation of $\mathbf{R}_1$ in closed-form. To overcome this limitation, in this section, we introduce[2] Theorem 1, which states a convergence (in the mean square sense) between the log-likelihood and its asymptotic version, and allows us to work with the log-likelihood in the frequency domain. Contrary to the log-likelihood, which is a function of the theoretical and estimated covariance matrices, the asymptotic log-likelihood is a function of the theoretical and estimated power spectral density (psd) matrices, and it can be seen as an extension of the Whittle's likelihood [10] for multivariate processes.

Before presenting Theorem 1, let us introduce some definitions. Consider an experiment producing $M$ ($M \geq L$) independent realizations of the data vector $\mathbf{z}$. Then, its log-likelihood is given by

$$\log p\left(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{R}\right) = -LNM \log \pi - M \log\det\left(\mathbf{R}\right) - M\text{tr}\left(\hat{\mathbf{R}}\mathbf{R}^{-1}\right),$$

and the asymptotic log-likelihood is

$$\log p\left(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{S}\left(e^{j\theta}\right)\right) = -LNM \log \pi - NM \int_{-\pi}^{\pi} \log\det\left(\mathbf{S}\left(e^{j\theta}\right)\right)\frac{d\theta}{2\pi} - NM \int_{-\pi}^{\pi} \text{tr}\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\mathbf{S}^{-1}\left(e^{j\theta}\right)\right)\frac{d\theta}{2\pi}, \quad (1)$$

where $\mathbf{R}$ is the theoretical block-Toeplitz covariance matrix, $\mathbf{S}\left(e^{j\theta}\right) = \mathcal{F}\left(\mathbf{R}[m]\right)$ is the theoretical psd matrix and their sample estimates are $\hat{\mathbf{R}} = \frac{1}{M}\sum_{i=0}^{M-1}\mathbf{z}_i\mathbf{z}_i^H$ and $\hat{\mathbf{S}}\left(e^{j\theta}\right) = \frac{1}{M}\sum_{i=0}^{M-1}\mathbf{x}_i\left(e^{j\theta}\right)\mathbf{x}_i^H\left(e^{j\theta}\right)$, where $\mathbf{x}_i\left(e^{j\theta}\right) = \frac{1}{\sqrt{N}}\sum_{n=0}^{N-1}\mathbf{x}_i[n]e^{-j\theta n}$.

*Theorem 1:* As $N \to \infty$, the asymptotic log-likelihood converges in the mean square sense to the true log-likelihood, i.e.,

$$\lim_{N\to\infty} E\left[\left|\frac{1}{N}\left[\log p\left(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{R}\right) - \log p\left(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{S}\left(e^{j\theta}\right)\right)\right]\right|^2\right] = 0.$$

As a direct consequence of Theorem 1, the hypothesis test asymptotically becomes

$$\mathcal{H}_1 : \mathbf{x}\left(e^{j\theta}\right) \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{S}_1\left(e^{j\theta}\right)\right),$$
$$\mathcal{H}_0 : \mathbf{x}\left(e^{j\theta}\right) \sim \mathcal{CN}\left(\mathbf{0}, \mathbf{S}_0\left(e^{j\theta}\right)\right),$$

where $\mathbf{S}_1\left(e^{j\theta}\right) = \mathbf{h}\left(e^{j\theta}\right)\mathbf{h}^H\left(e^{j\theta}\right) + \mathbf{\Sigma}\left(e^{j\theta}\right)$, $\mathbf{S}_0\left(e^{j\theta}\right) = \mathbf{\Sigma}\left(e^{j\theta}\right)$, $\mathbf{h}\left(e^{j\theta}\right)$ is the Fourier transform of the SIMO channel and $\mathbf{\Sigma}\left(e^{j\theta}\right)$ is a diagonal matrix which contains the psd of the noises. Therefore, under $\mathcal{H}_1$ the psd matrix is a rank-one matrix plus a diagonal term and under $\mathcal{H}_0$ it is only a diagonal matrix.

## IV. GLRT for iid. noises

In this section, we derive the asymptotic GLRT for equally distributed noises, i.e., $\mathbf{\Sigma}\left(e^{j\theta}\right) = S_v\left(e^{j\theta}\right)\mathbf{I}$, where $S_v\left(e^{j\theta}\right)$ is the unknown psd of the noise and $\mathbf{I}$ is the $L \times L$ identity matrix. The log-GLRT is given by

$$l = \max_{S_v\left(e^{j\theta}\right)} \log p_0\left(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; S_v\left(e^{j\theta}\right)\right) - \max_{\mathbf{h}\left(e^{j\theta}\right), S_v\left(e^{j\theta}\right)} \log p_1\left(\mathbf{z}_0, \dots, \mathbf{z}_{M-1}; \mathbf{h}\left(e^{j\theta}\right), S_v\left(e^{j\theta}\right)\right), \quad (2)$$

and we need to find the ML estimates of the unknown parameters under each hypothesis. Under $\mathcal{H}_0$, only $S_v\left(e^{j\theta}\right)$ is unknown, and it is easy to show that its ML estimate is $\hat{S}_v\left(e^{j\theta}\right) = \frac{1}{L}\text{tr}\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)$. Now, under $\mathcal{H}_1$, we should find the ML estimates of $S_v\left(e^{j\theta}\right)$ and $\mathbf{h}\left(e^{j\theta}\right)$. Before proceeding, we shall apply the determinant and the matrix inversion lemmas to $\mathbf{S}_1\left(e^{j\theta}\right)$, which, for this particular case, read as follows

$$\det\left(\mathbf{S}_1\left(e^{j\theta}\right)\right) = \left(1 + \frac{\mathbf{h}^H\left(e^{j\theta}\right)\mathbf{h}\left(e^{j\theta}\right)}{S_v\left(e^{j\theta}\right)}\right)S_v^L\left(e^{j\theta}\right), \quad (3)$$

$$\mathbf{S}_1^{-1}\left(e^{j\theta}\right) = \frac{1}{S_v\left(e^{j\theta}\right)}\mathbf{I} - \frac{\mathbf{h}\left(e^{j\theta}\right)\mathbf{h}^H\left(e^{j\theta}\right)}{S_v^2\left(e^{j\theta}\right) + S_v\left(e^{j\theta}\right)\left\|\mathbf{h}\left(e^{j\theta}\right)\right\|^2}. \quad (4)$$

Substituting (3) and (4) in the asymptotic log-likelihood (1), and after some tedious but straightforward algebra, the ML estimates are given by

$$\hat{S}_v\left(e^{j\theta}\right) = \frac{1}{L-1}\sum_{i=2}^{L}\lambda_i\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right),$$

$$\hat{\mathbf{h}}\left(e^{j\theta}\right) = \sqrt{\lambda_1\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right) - \hat{S}_v\left(e^{j\theta}\right)} \cdot \text{ev}_1\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right),$$

where $\text{ev}_i\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)$ denotes the unit-norm eigenvector associated to the $i$-th largest eigenvalue of $\hat{\mathbf{S}}\left(e^{j\theta}\right)$, which is denoted by $\lambda_i\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)$. Substituting in (2) the ML estimates under both

[2]For the sake of space, the proof of the theorem will be presented in a forthcoming journal version of this paper.

hypotheses and omitting some constant terms, the log-GLRT is finally given by

$$l = (L-1)\int_{-\pi}^{\pi} \log\left[1 - \frac{\lambda_{\text{MAX}}\left(e^{j\theta}\right)}{\text{tr}\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)}\right]\frac{d\theta}{2\pi} + \int_{-\pi}^{\pi} \log \frac{\lambda_{\text{MAX}}\left(e^{j\theta}\right)}{\text{tr}\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)}\frac{d\theta}{2\pi}, \quad (5)$$

where $\lambda_{\text{MAX}}\left(e^{j\theta}\right) = \lambda_1\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)$ is the largest eigenvalue of $\hat{\mathbf{S}}\left(e^{j\theta}\right)$ and $\text{tr}\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)$ denotes the trace of $\hat{\mathbf{S}}\left(e^{j\theta}\right)$. Interestingly, the GLRT in (5) nicely extends the result of [8] to time series with temporal structure. For time series without temporal structure, [8] shows that the GLRT is

$$l = \frac{\lambda_1\left(\hat{\mathbf{R}}[0]\right)}{\text{tr}\left(\hat{\mathbf{R}}[0]\right)}, \quad (6)$$

where

$$\hat{\mathbf{R}}[0] = \frac{1}{NM}\sum_{i=0}^{M-1} \mathbf{X}_i\mathbf{X}_i^H.$$

Therefore, the derived GLRT shows how the frequency-dependent GLRTs obtained in [8] must be merged into a single test statistic when the vector-valued random variable is replaced by a vector-valued time series with temporal correlation.

## V. GLRT FOR NON-IID NOISES IN THE LOW SNR REGIME

Let us now consider noises with different power spectral densities. That is, the psd matrix $\boldsymbol{\Sigma}\left(e^{j\theta}\right)$ is still diagonal, but not necessarily proportional to the identity matrix. The log-GLRT is now given by

$$l = \max_{\boldsymbol{\Sigma}\left(e^{j\theta}\right)} \log p_0\left(\mathbf{z}_0,\ldots,\mathbf{z}_{M-1};\boldsymbol{\Sigma}\left(e^{j\theta}\right)\right) - \max_{\mathbf{h}\left(e^{j\theta}\right),\boldsymbol{\Sigma}\left(e^{j\theta}\right)} \log p_1\left(\mathbf{z}_0,\ldots,\mathbf{z}_{M-1};\mathbf{h}\left(e^{j\theta}\right),\boldsymbol{\Sigma}\left(e^{j\theta}\right)\right).$$

Under $\mathcal{H}_0$, only $\boldsymbol{\Sigma}\left(e^{j\theta}\right)$ is unknown and its ML estimate is given by $\hat{\boldsymbol{\Sigma}}\left(e^{j\theta}\right) = \hat{\mathbf{D}}\left(e^{j\theta}\right) = \text{diag}\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)$, where $\text{diag}\left(\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)$ denotes a diagonal matrix formed from the main diagonal of $\hat{\mathbf{S}}\left(e^{j\theta}\right)$.

The ML estimation of the unknown parameters under $\mathcal{H}_1$ does not have, in general, a closed-form solution. However, as we will show, it is possible to find a closed-form solution for the ML estimates in the low signal-to-noise (SNR) regime. Although this could seem quite restrictive, the low SNR region is of particular interest in cognitive radio, which makes the detector useful.

Let us consider first that the psd of the noise processes are known and define[3]

$$\tilde{\mathbf{h}}\left(e^{j\theta}\right) = \frac{\boldsymbol{\Sigma}^{-1}\left(e^{j\theta}\right)\mathbf{h}\left(e^{j\theta}\right)}{\sqrt{\mathbf{h}^H\left(e^{j\theta}\right)\boldsymbol{\Sigma}^{-1}\left(e^{j\theta}\right)\mathbf{h}\left(e^{j\theta}\right)}}, \quad (7)$$

$$\alpha\left(e^{j\theta}\right) = \mathbf{h}^H\left(e^{j\theta}\right)\boldsymbol{\Sigma}^{-1}\left(e^{j\theta}\right)\mathbf{h}\left(e^{j\theta}\right), \quad (8)$$

where $\tilde{\mathbf{h}}^H\left(e^{j\theta}\right)\boldsymbol{\Sigma}\left(e^{j\theta}\right)\tilde{\mathbf{h}}\left(e^{j\theta}\right) = 1$. Substituting (7) and (8) in the log-likelihood and taking into account the determinant and the matrix inversion lemmas, it is straightforward to show that the ML estimate of $\tilde{\mathbf{h}}\left(e^{j\theta}\right)$ is the (properly) normalized eigenvector associated to $\lambda_1\left(\boldsymbol{\Sigma}^{-1}\left(e^{j\theta}\right)\hat{\mathbf{S}}\left(e^{j\theta}\right)\right)$, and the ML estimate of $\alpha\left(e^{j\theta}\right)$ is given by $\hat{\alpha}\left(e^{j\theta}\right) = \lambda_1\left(\boldsymbol{\Sigma}^{-1}\left(e^{j\theta}\right)\hat{\mathbf{S}}\left(e^{j\theta}\right)\right) - 1$. Now, we should find the ML estimate of $\boldsymbol{\Sigma}\left(e^{j\theta}\right)$, and as previously pointed out, we shall consider the low SNR region. In this situation, it is easy to see that $\mathbf{S}_1\left(e^{j\theta}\right)$ is dominated by $\boldsymbol{\Sigma}\left(e^{j\theta}\right)$, and therefore, the ML estimate of $\boldsymbol{\Sigma}\left(e^{j\theta}\right)$ is approximately given by $\hat{\boldsymbol{\Sigma}}\left(e^{j\theta}\right) \approx \hat{\mathbf{D}}\left(e^{j\theta}\right)$. Finally, taking into account the ML estimates under both hypotheses and omitting some constant terms, the GLRT is approximately given by

$$l \approx \int_{-\pi}^{\pi} \log \lambda_{\text{MAX}}\left(e^{j\theta}\right)\frac{d\theta}{2\pi} - \int_{-\pi}^{\pi} \lambda_{\text{MAX}}\left(e^{j\theta}\right)\frac{d\theta}{2\pi}. \quad (9)$$

where $\lambda_{\text{MAX}}\left(e^{j\theta}\right)$ is the largest eigenvalue of $\hat{\mathbf{D}}^{-1}\left(e^{j\theta}\right)\hat{\mathbf{S}}\left(e^{j\theta}\right)$. Similar to the GLRT derived in Section IV, the GLRT given by (9) extends the results of [9] to vector-valued time series with temporal structure. Specifically, the detector proposed in [9] is

$$l = \lambda_1\left(\hat{\mathbf{D}}^{-1}[0]\hat{\mathbf{R}}[0]\right), \quad (10)$$

where $\hat{\mathbf{D}}[0] = \text{diag}\left(\hat{\mathbf{R}}[0]\right)$.

### A. Relationship with the GCS-MAX

In this subsection, we show the relationship between the GLRT in (9) and a recently proposed generalization of coherence spectrum (GCS) for more than two signals [11]. We shall start by defining the matrix

$$\hat{\mathbf{C}}\left(e^{j\theta}\right) = \hat{\mathbf{D}}^{-1/2}\left(e^{j\theta}\right)\hat{\mathbf{S}}\left(e^{j\theta}\right)\hat{\mathbf{D}}^{-1/2}\left(e^{j\theta}\right),$$

which contains all the estimated pairwise complex coherence spectra. Thus, the GCS-MAX is defined as follows [11]

$$\hat{\gamma}\left(e^{j\theta}\right) = \frac{1}{L}\left[\lambda_1\left(\hat{\mathbf{C}}\left(e^{j\theta}\right)\right) - 1\right].$$

Taking into account that the eigenvalues of $\hat{\mathbf{C}}\left(e^{j\theta}\right)$ are equal to the eigenvalues of $\hat{\mathbf{D}}^{-1}\left(e^{j\theta}\right)\hat{\mathbf{S}}\left(e^{j\theta}\right)$, the GLRT given by (9) can be rewritten as

$$l \approx \int_{-\pi}^{\pi} \log\left[L\hat{\gamma}\left(e^{j\theta}\right) + 1\right]\frac{d\theta}{2\pi} - L\int_{-\pi}^{\pi} \hat{\gamma}\left(e^{j\theta}\right)\frac{d\theta}{2\pi}. \quad (11)$$

The GCS-MAX was previously proposed as an statistic for the multiple-channel detection problem in [12]. However, in [12], the GCS-MAX was directly integrated, instead of using (11), which yields the GLRT.

## VI. SIMULATION RESULTS

In this section, the performance of the proposed detectors (eq. (5) and eq. (9)) is illustrated by means of some simulation results. For comparison purposes, we have also evaluated the following detectors:

- The GLRT for white time series, with equal (eq. (6)) and different (eq. (10)) noise distributions.
- The GLRT for time series with temporal structure and an unknown spatial structure [7], which is given by

$$l = \int_{-\pi}^{\pi} \log\det(\hat{\mathbf{C}}(e^{j\theta}))\frac{d\theta}{2\pi}.$$

In the first example, we have used an OFDM-modulated DVB-T signal[4] with a bandwidth of 7.61 MHz. The signal undergoes propagation through a typical urban area channel: spatially uncorrelated frequency-selective Rayleigh fading channel with exponential power delay profile, unit power and delay spread of 0.779 $\mu$sec [13]. At the spectrum monitor, the signals are downconverted and asynchronously sampled at 16 MHz. The additive noises at each antenna

[3]We can easily obtain $\mathbf{h}\left(e^{j\theta}\right)$ from $\tilde{\mathbf{h}}\left(e^{j\theta}\right)$ and $\alpha\left(e^{j\theta}\right)$.

[4]8K mode, 64-QAM, guard interval $1/4$ and inner code rate $2/3$.

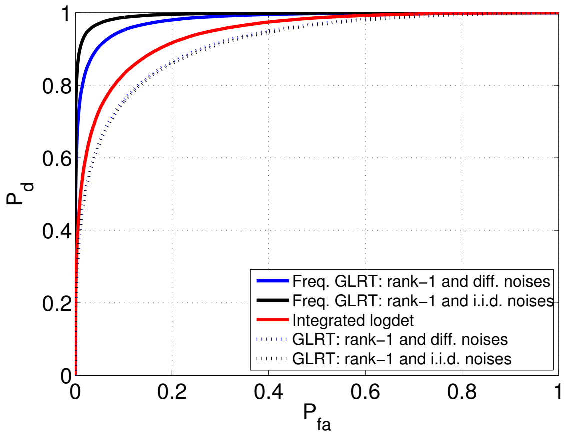


Fig. 1. ROC for the example with i.i.d. noises.

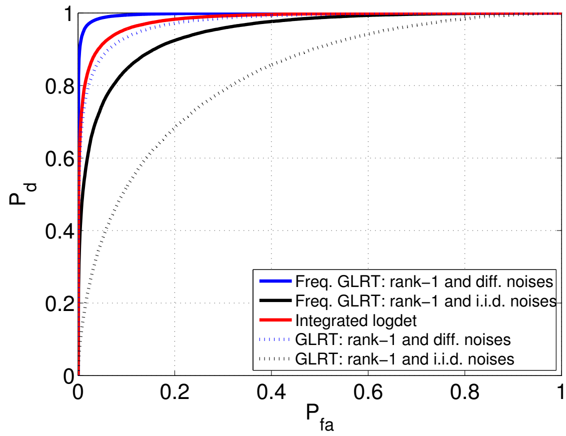


Fig. 2. ROC for the example with different distributed noises.

are independent zero-mean and complex white Gaussian processes with common variance $\sigma^2$, and the common SNR for all antennas is defined as $\text{SNR(dB)} = 10\log_{10}(1/\sigma^2)$. For the frequency-domain detectors, the integrals are computed via the trapezoidal method with $K = 256$ equispaced points of the estimated psd matrix.

Fig. 1 shows the receiver operational characteristic (ROC) curve for this example. We have considered $L = 5$ antennas, $M = 10$ realizations of length $N = 100$, and a SNR $= -10$ dB. As expected, the best results are obtained by the GLRT given by (5), which exploits the previous knowledge about the identical distributions of the noise. Additionally, we can see that the GLRT given by (9) provides only a slight degradation because it does not exploit the fact that the noises' psd are equal. Moreover, it is clear that the detectors exploiting the time structure of the processes outperform their counterparts for white processes and, obviously, the worst results are obtained by the detector that does not exploit the rank-one structure. The increased performance of the frequency domain detectors comes at the expense of an increased computational complexity, which is approximately $K$ times bigger than their time domain counterparts.

In the second example, all the parameters remain the same with the exception of the noise psd. In this case, each noise process has variance $a_i\sigma^2$ where $a_i,\ i = 1,\ldots,L$, is a uniform random variable between 1 and the maximum noise mismatch (we will consider a maximum mismatch of 3 dB), whereas the SNR is still defined as in the previous example[5] and its value is SNR $= -6$ dB. Fig. 2 shows the ROC curve for this example where the most remarkable fact is that the performance of the GLRT detectors that assume equally distributed noises degrades noticeable, even for this small value of noise mismatch.

## VII. Conclusion

In this work we have presented a new multiantenna detector for spectrum sensing in cognitive radio based on the asymptotic likelihood. We have derived the GLRT for detecting a wideband rank-one signal under spatially uncorrelated noise with equal or different power spectral densities. Moreover, simulations have shown the strong impact of incorrectly assuming equally distributed noises. Thus, taking into account that, due to imperfect analog components and calibration errors, different noises are commonly found in practice, we encourage the practical application of the GLRT for noises with different distributions. Finally, it is important to point out that this detector does not require synchronization at any level with the primary signal, and no assumptions have been made about the spectra of the primary signal and/or the noise.

[5]Obviously, the actual value of the SNR will be lower.

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