# MULTIANTENNA SPECTRUM SENSING: DETECTION OF SPATIAL CORRELATION AMONG TIME-SERIES WITH UNKNOWN SPECTRA

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**ABSTRACT**

One of the key problems in cognitive radio (CR) is the detection of primary activity in order to determine which parts of the spectrum are available for opportunistic access. This detection task is challenging, since the wireless environment often results in very low SNR conditions. Moreover, calibration errors and imperfect analog components at the CR spectral monitor result in uncertainties in the noise spectrum, making the problem more difficult. In this work, we present a new multiantenna detector which is based on the fact that the observation noise processes are spatially uncorrelated, whereas any primary signal present should result in spatial correlation. In particular, we derive the generalized likelihood ratio test (GLRT) for this problem, which is given by the quotient between the determinant of the sample covariance matrix and the determinant of its block-diagonal version. For stationary processes the GLRT tends asymptotically to the integral of the logarithm of the Hadamard ratio of the estimated power spectral density matrix. Additionally, we present an approximation of the frequency domain detector in the low SNR regime, which results in computational savings. The performance of the proposed detectors is evaluated by means of numerical simulations, showing important advantages over existing detectors.

***Index Terms*—** Cognitive radio, multiple-channel signal detection, generalized likelihood ratio test, Hadamard ratio, coherence spectrum.

## 1. INTRODUCTION

In the last years the cognitive radio (CR) paradigm has emerged as a key technology to improve spectrum usage [1]. The basic idea behind CR is the opportunistic access of some users (secondary users) to the wireless channel when the licensed (primary) users are not transmitting. Therefore, any CR system necessarily relies on a spectrum sensing device for determining which parts of the spectrum band are available (spectrum holes). Even when a spectrum hole is found and exploited, secondary users must periodically check whether it has been reclaimed by the primary network, in which case the spectrum hole must be quickly vacated.

Detection of primary users in CR is a challenging problem, since fading and shadowing may result in very weak received primary signals. This means that the spectrum monitor must operate in very low SNR environments preventing synchronization to and/or decoding of these signals, even if the modulation format and parameters of primary transmitters were known. A number of detectors have been proposed for CR applications, see [2] and references therein. Perhaps the most popular (and computationally cheapest) one is the energy detector (ED), which does not require any *a priori* information about the primary system and does not need any sort of synchronization. The main drawback of the ED resides in its sensitivity to uncertainties in the background noise power, which may result in undetectable primary signals if the SNR is below certain level, *even as the observation time goes to infinity* [3]. Alternative approaches to the ED exploit some features of primary signals, such as cyclostationarity or the presence of pilots. However, these methods are sensitive to synchronization errors [4], unavoidable in low SNR conditions.

Another way to improve the detection performance of spectrum monitors is to use multiple antennas. Intuitively, the presence of any primary signal should result in spatial correlation in the observations; a feature that can be used for detection since the noise processes at different antennas can be safely assumed statistically independent. A multiantenna ED extension that also exploits knowledge of the primary spectral emission mask was proposed in [5], but this scheme remains sensitive to noise uncertainty. The multiantenna detector suggested in [6] does not need knowledge of the noise variance, but it implicitly assumes that the noise processes are white and with the same power at all antennas. In practice, calibration errors become unavoidable, and thus any deviation from these assumptions will result in performance degradation.

In this work we propose a multiantenna detector in which no assumptions are made about the primary signal nor the spectral properties of the noise. Rather, it is exclusively based on the assumption that, in the absence of primary transmissions, the observations are spatially uncorrelated. We derive the generalized likelihood ratio test (GLRT) for the block-diagonal structure of the space-time covariance matrix, which is asymptotically approximated by the integral of the log of the Hadamard ratio of the estimated power spectral density (psd) matrix. In the low SNR regime, of particular interest in CR applications, a computationally cheaper approximation of the frequency domain detector can be derived. The benefits of the proposed detectors are illustrated by means of some numerical simulations.

## 2. PROBLEM FORMULATION

We address the problem of detecting the presence of a primary user in a cognitive radio node equipped with $L$ antennas, without any prior knowledge about the primary transmission, the wireless channel, or the noise processes (beyond spatial independence). In particular, we test the covariance structure of the vector-valued time series $\{\mathbf{x}[n],\ n = 0, \pm1, \dots\}$, where $\mathbf{x}[n] = [x_1[n], \dots, x_L[n]]^T$ is a vector of measurements at time $n$, or equivalently, $\{x_i[n]\}$ is the time series at the $i$-th antenna. The detection problem is given by

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{x}[n] = \mathbf{s}[n] + \mathbf{v}[n], \quad && n = 0, \dots, N-1, \\ \mathcal{H}_0 &: \mathbf{x}[n] = \mathbf{v}[n], && n = 0, \dots, N-1, \end{aligned}$$

where $\mathbf{s}[n]$ is the vector with the samples of the primary signal at the $L$ antennas, and at time $n$; and $\mathbf{v}[n] = [v_1[n], \dots, v_L[n]]^T$ is the additive noise vector, which is assumed to be zero-mean circular complex Gaussian and spatially white, i.e., $E\{v_i[n]v_j^*[k]\} = 0$ for $i \neq j$ and all $n$, $k$. No assumptions are made on the *temporal* correlation of the noise processes, $E\{v_i[n]v_i^*[k]\}$.

Let us define the data matrix

$$\mathbf{X} = \begin{bmatrix} x_1[0] & x_1[1] & \dots & x_1[N-1] \\ x_2[0] & x_2[1] & \dots & x_2[N-1] \\ \vdots & \vdots & \ddots & \vdots \\ x_L[0] & x_L[1] & \dots & x_L[N-1] \end{bmatrix} = \begin{bmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_L^T \end{bmatrix},$$

where the $i$-th row, $\mathbf{x}_i^T = [x_i[0], x_i[1], \dots, x_i[N-1]]$, contains $N$-samples of the $i$-th time series $\{x_i[n]\}$, and the $n$-th column is the $n$-th sample of the vector-valued time series $\{\mathbf{x}[n]\}$. The vector $\mathbf{z} = \text{vec}\left(\mathbf{X}^T\right)$ stacks the columns of $\mathbf{X}^T$, and its covariance matrix is

$$\mathbf{R} = E\left[\mathbf{z}\mathbf{z}^H\right] = \begin{bmatrix} \mathbf{R}_{11} & \mathbf{R}_{21}^H & \dots & \mathbf{R}_{L1}^H \\ \mathbf{R}_{21} & \mathbf{R}_{22} & \dots & \mathbf{R}_{L2}^H \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{R}_{L1} & \mathbf{R}_{L2} & \dots & \mathbf{R}_{LL} \end{bmatrix} \in \mathbb{C}^{LN \times LN}.$$

The covariance matrices $\mathbf{R}_{ik} = E\left[\mathbf{x}_i\mathbf{x}_k^H\right],\ 1 \leq i, k \leq L$ capture all space-time second-order information about the random vectors $\{\mathbf{x}_i\}_{i=1}^L$.

In order to proceed, we need the distribution of $\{\mathbf{x}[n]\}$ under $\mathcal{H}_1$. We take it to be zero-mean, circular complex Gaussian. In addition to resulting in tractable models and useful detectors, this assumption is reasonable if the primary network employs orthogonal frequency division multiplexing (OFDM) as modulation format. Thus, the hypothesis testing problem becomes

$$\begin{aligned} \mathcal{H}_1 &: \mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_1), \\ \mathcal{H}_0 &: \mathbf{z} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R}_0), \end{aligned}$$

where $\mathcal{CN}(\mathbf{0}, \mathbf{R}_l)$ denotes the complex Gaussian distribution with zero mean and covariance $\mathbf{R}_l$. Under $\mathcal{H}_0$, $\mathbf{R}_0$ is an unknown positive definite block-diagonal matrix, i.e. $\mathbf{R}_0 \in \mathfrak{R}_0$, where $\mathfrak{R}_0 = \{\mathbf{R} \mid \mathbf{R} = \text{diag}(\mathbf{R}_{11}, \dots, \mathbf{R}_{LL})\}$, with the only constraint that $\mathbf{R}_{ii}$ is Hermitian positive definite, and under $\mathcal{H}_1$, $\mathbf{R}_1 \in \mathfrak{R}_1$, where $\mathfrak{R}_1$ is the set of unknown positive definite covariance matrices with no temporal or spatial structure, since we do not use any prior information about the primary signals. The block-diagonal structure of the covariance matrix under the null hypothesis is due to spatial uncorrelation of the noise.

## 3. DERIVATION OF THE GLRT

Let us assume an experiment producing $M$ independent realizations of the data matrix $\mathbf{X}$, or equivalently $\mathbf{z}$. The joint probability density function (pdf) for these measurements is the product of the pdfs, and is given by

$$p(\mathbf{z}[0], \dots, \mathbf{z}[M-1]; \mathbf{R}) = \prod_{n=0}^{M-1} p(\mathbf{z}[n]; \mathbf{R}) = \\ = \frac{1}{\pi^{LNM} \det(\mathbf{R})^M} \exp\left\{-M\text{Tr}\left(\mathbf{R}^{-1}\hat{\mathbf{R}}\right)\right\},$$

where $\hat{\mathbf{R}}$ is the sample covariance matrix given by,

$$\hat{\mathbf{R}} = \frac{1}{M}\sum_{n=0}^{M-1} \mathbf{z}[n]\mathbf{z}^H[n] = \begin{bmatrix} \hat{\mathbf{R}}_{11} & \hat{\mathbf{R}}_{21}^H & \dots & \hat{\mathbf{R}}_{L1}^H \\ \hat{\mathbf{R}}_{21} & \hat{\mathbf{R}}_{22} & \dots & \hat{\mathbf{R}}_{L2}^H \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\mathbf{R}}_{L1} & \hat{\mathbf{R}}_{L2} & \dots & \hat{\mathbf{R}}_{LL} \end{bmatrix},$$

and $\hat{\mathbf{R}}_{ik} \in \mathbb{C}^{N \times N}$ is the $\{i, k\}$-th block of $\hat{\mathbf{R}}$, which represents the estimated cross-covariance matrix between the $N$-sample windows of the $i$-th and $k$-th time series.

To solve our hypothesis testing problem, we will use the generalized likelihood ratio test (GLRT). Although it is known that the GLRT is not optimal in the Neyman-Pearson sense, it provides good performance [7]. The GLRT for $\mathcal{H}_0 : \mathbf{R} \in \mathfrak{R}_0$ vs. $\mathcal{H}_1 : \mathbf{R} \in \mathfrak{R}_1$ is based on the generalized likelihood ratio (GLR) [7]

$$\lambda = \frac{\max\limits_{\mathbf{R} \in \mathfrak{R}_0} p(\mathbf{z}[0], \dots, \mathbf{z}[M-1]; \mathbf{R})}{\max\limits_{\mathbf{R} \in \mathfrak{R}_1} p(\mathbf{z}[0], \dots, \mathbf{z}[M-1]; \mathbf{R})} \\ = \det\left(\hat{\mathbf{R}}_0^{-1}\hat{\mathbf{R}}_1\right)^M \exp\left\{-M\text{tr}\left[\left(\hat{\mathbf{R}}_0^{-1} - \hat{\mathbf{R}}_1^{-1}\right)\hat{\mathbf{R}}\right]\right\},$$

where $\hat{\mathbf{R}}_1$ and $\hat{\mathbf{R}}_0$ are the maximum likelihood estimates of $\mathbf{R}$ under hypotheses $\mathcal{H}_1$ and $\mathcal{H}_0$, respectively.

Now, we will obtain the ML estimates of the covariance matrices under both hypotheses, for which we need to assume $M \geq N$. As previously pointed out, under $\mathcal{H}_0$ the correlation matrix $\mathbf{R}$ is block-diagonal, with the only constraint that $\mathbf{R}_{ii}$ is Hermitian non-negative definite. That is, we force spatial uncorrelatedness but do not force temporal stationarity. Then, it is easy to show that the ML estimate of $\hat{\mathbf{R}}_0$ is $\hat{\mathbf{R}}_0 = \text{diag}\left(\hat{\mathbf{R}}_{11}, \hat{\mathbf{R}}_{22}, \dots, \hat{\mathbf{R}}_{LL}\right)$.

For $\mathcal{H}_1$ we take $\mathfrak{R}_1$ to be the set of matrices $\mathbf{R}$, with no temporal or spatial structure imposed, with the only constraint being that $\mathbf{R}$ is an Hermitian non-negative definite matrix. Then, the ML estimate of $\mathbf{R}_1$ is given by $\hat{\mathbf{R}}_1 = \hat{\mathbf{R}}$. Taking the ML estimates into account, the GLRT is

$$\lambda^{\frac{1}{NM}} = \det\left(\hat{\mathbf{R}}_0^{-1}\hat{\mathbf{R}}_1\right)^{\frac{1}{N}} = \frac{\left[\det\left(\hat{\mathbf{R}}\right)\right]^{\frac{1}{N}}}{\left[\prod\limits_{i=1}^{L} \det\left(\hat{\mathbf{R}}_{ii}\right)\right]^{\frac{1}{N}}}, \tag{1}$$

which is a special case of a general result in [7]. Specifically, the GLRT is a *generalized* Hadamard ratio. Interestingly, this statistic is invariant to independent linear transformations of the time-series, including any arbitrary filtering of the sequences $\{x_i[n]\}$.

### 3.1. Frequency Domain Detector

The time domain detector in (1) was derived without any stationary assumption. When the time series $\{x_i[n]\}, i = 1, \dots, L$, are jointly stationary random vectors whose dimensions increase without bound (jointly stationary time series) and following an argument along the lines of [8], the limiting form of (1) ($L$ fixed and $N \to \infty$) may be *approximated*[1] by

$$l = \lambda^{\frac{1}{NM}} = \frac{\exp\left\{ \int_{-\pi}^{\pi} \log\det\left[\hat{\mathbf{S}}\left(e^{j\theta}\right)\right] \frac{d\theta}{2\pi} \right\}}{\exp\left\{ \int_{-\pi}^{\pi} \log\left[\prod_{i=1}^{L} \hat{S}_{ii}\left(e^{j\theta}\right)\right] \frac{d\theta}{2\pi} \right\}}, \qquad (2)$$

where $\hat{\mathbf{S}}(e^{j\theta})$ is a standard quadratic estimator of the psd matrix, averaged over $M$ realizations

$$\hat{\mathbf{S}}(e^{j\theta}) = \begin{bmatrix} \hat{S}_{11}(e^{j\theta}) & \hat{S}_{21}^*(e^{j\theta}) & \dots & \hat{S}_{L1}^*(e^{j\theta}) \\ \hat{S}_{21}(e^{j\theta}) & \hat{S}_{22}(e^{j\theta}) & \dots & \hat{S}_{L2}^*(e^{j\theta}) \\ \vdots & \vdots & \ddots & \vdots \\ \hat{S}_{L1}(e^{j\theta}) & \hat{S}_{L2}(e^{j\theta}) & \dots & \hat{S}_{LL}(e^{j\theta}) \end{bmatrix}.$$

Therefore, (2) can be rewritten as

$$l = \exp\left\{ \int_{-\pi}^{\pi} \log\left[\frac{\det\left[\hat{\mathbf{S}}\left(e^{j\theta}\right)\right]}{\prod_{i=1}^{L} \hat{S}_{ii}\left(e^{j\theta}\right)}\right] \frac{d\theta}{2\pi} \right\}, \qquad (3)$$

i.e., the GLRT in the frequency domain can be approximated by the integral over the Nyquist band of the logarithm of a Hadamard ratio. Finally, we must point out that in the case of $L = 2$ time series the term inside the logarithm is just a function of the magnitude squared coherence (MSC) spectrum [8].

### 3.2. Low SNR Approximation

In cognitive radio, the most interesting case is the low SNR regime. In this scenario, and following the ideas of [9], the statistic in (3) can be approximated by

$$l \approx \exp\left\{ -\frac{1}{2}\int_{-\pi}^{\pi} \left\|\hat{\mathbf{C}}(e^{j\theta})\right\|_F^2 \frac{d\theta}{2\pi} + \frac{L}{2} \right\}, \qquad (4)$$

where $\hat{\mathbf{C}}(e^{j\theta}) = \hat{\mathbf{D}}(e^{j\theta})^{-1/2}\hat{\mathbf{S}}(e^{j\theta})\hat{\mathbf{D}}(e^{j\theta})^{-1/2}$ and $\hat{\mathbf{D}}(e^{j\theta})$ is a diagonal matrix formed from the main diagonal of $\hat{\mathbf{S}}(e^{j\theta})$. This approximation, which can be seen as a generalization of [9] to vector-valued time series, allows us to simplify the detector in the low SNR regime, and it also results in a more robust test statistic when the number of available samples is small.

## 4. SIMULATION RESULTS

In this section, we present some simulation results to illustrate the performance of the proposed detectors (eq. (3) and eq. (4)) and compare it to that of the following detectors:

- The energy detector (ED) using $LN$ samples per realization (the total number of samples is therefore $MLN$).
- The GLRT for white time series [9], which is equivalent to the generalized coherence (GC) proposed in [10] and is given by $1 - \det(\hat{\mathbf{C}}[0])$, where $\hat{\mathbf{C}[0]} = \hat{\mathbf{D}}[0]^{-1/2}\hat{\mathbf{R}}[0]\hat{\mathbf{D}}[0]^{-1/2}$, is the $L \times L$ spatial coherence matrix in the time domain,

  $$\hat{\mathbf{R}}[0] = \frac{1}{NM}\sum_{n=0}^{M-1} \mathbf{X}[n]\mathbf{X}^H[n],$$

  and $\hat{\mathbf{D}}[0]$ is a diagonal matrix formed from the main diagonal of $\hat{\mathbf{R}}[0]$.
- A modification of the detector [11] to handle noises with different powers at each antenna. The detector is based on the ratio of largest to smallest eigenvalues of the spatial coherence matrix $\hat{\mathbf{C}[0]}$.

For the simulations, we have used an OFDM-modulated DVB-T signal[2] with a bandwidth of 7.61 MHz. The signal undergoes propagation through a spatially uncorrelated frequency-selective Rayleigh fading channel with exponential power delay profile and unit power; at the spectrum monitor, it is downconverted and asynchronously sampled at 16 MHz. The additive noises at each antenna are generated by filtering independent zero-mean and complex white Gaussian processes with common variance $\sigma^2$ with finite impulse response (FIR) filters with 4 i.i.d. random taps distributed as $a_i[n] \sim \mathcal{CN}(0, 1/4), n = 0, \dots, 3;\ i = 1, \dots, L$, and the common SNR for all antennas is defined as SNR(dB) $= 10\log_{10}(1/\sigma^2)$.

Figures 1 and 2 show the receiver operating characteristic (ROC) curve for a typical rural area (delay spread of 0.097 $\mu$sec) and for a typical urban area (delay spread of 0.779 $\mu$sec) [12]. The remaining parameters are: $L = 3$ antennas, $N = 100$ samples[3], the number of realizations is $M = 10$, the signal-to-noise ratio is SNR $= 0$ dB and the psd matrix is estimated using the Welch's approach. As can be seen in the figures, the proposed detectors present the best results, mainly for the most selective channel (Fig. 2), which indicates that exploiting the frequency structure of the time series significantly improves the performance of the detectors. These examples also show that the Frobenius norm approximation (denoted as F-GLRT in the figures) presents good results, and it even outperforms the logdet detector in some cases. Obviously, the GC and the detector based on the eigenvalue spread perform poorly because they were designed for temporarily white processes, and never intended for correlated time series.

Finally, Fig. 3 shows the miss probability as a function of the SNR for a fixed value of the false alarm probability ($p_{FA} = 0.01$) using the same parameters of the second example. Contrary to the energy detector, the threshold of the proposed detectors does not depend on the actual value of the SNR. Therefore, following the ideas of [13], it can be calculated in advance by simulations. In the figure, we can see that the proposed detectors obtain the highest slopes and that the Frobenius approximation performs well for low and moderate SNRs.

## 5. CONCLUSIONS

In this work we have presented a new multiantenna detector for spectrum sensing in cognitive radio. This detector does not require synchronization at any level with the primary signal, and is based on the fact that under the noise-only hypothesis, the observations should be

[1] Notice that the ML estimates of the covariance matrices in (1) are not Toeplitz, in general; and consequently (2) is just an approximation of the asymptotic GLRT for stationary processes.

[2] 8K mode, 64-QAM, guard interval $1/4$ and inner code rate $2/3$.

[3] For the energy detector the total number of samples is $MLN = 3000$.

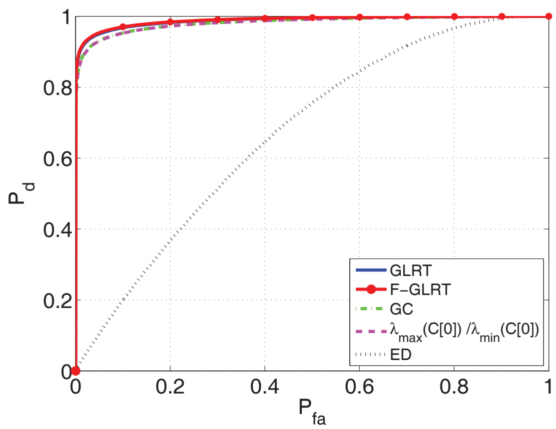


**Fig. 1**. ROC for the rural area. We have considered $L = 3$ antennas, $M = 10$ realizations of length $N = 100$ and the SNR $= 0$ dB.

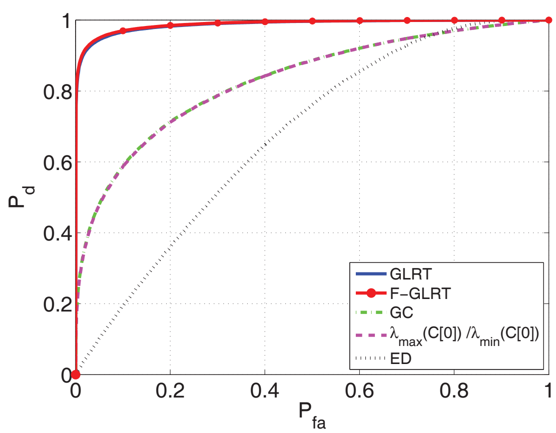


**Fig. 2**. ROC for the urban area. We have considered $L = 3$ antennas, $M = 10$ realizations of length $N = 100$ and the SNR $= 0$ dB.

spatially uncorrelated. The GLRT, and a frequency domain approximation were derived under a Gaussian signal model. Since no assumptions are made on the power and spectra (nor even stationarity) of the signal and/or the noise, this scheme is robust to uncertainties in this regard, commonly found in practice due to imperfect analog components and calibration errors.

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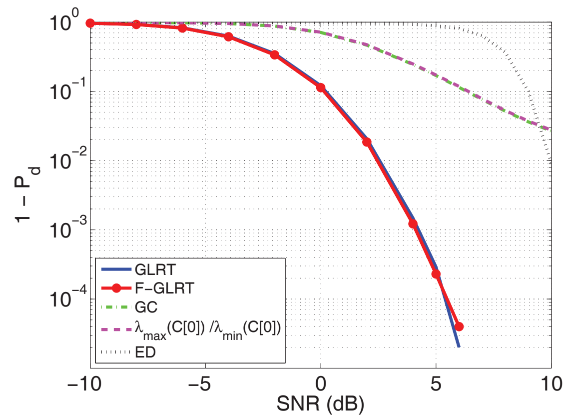


**Fig. 3**. Detection performance for the urban area as a function of the SNR for $p_{FA} = 0.01$.

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